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Adaptive robust dynamic surface control with composite adaptation laws

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SUMMARY

This paper focuses on an adaptive robust dynamic surface control (ARDSC) with composite adaptation laws (CAL) for a class of uncertain nonlinear systems in semi-strict feedback form. A simple and effective controller has been obtained by introducing dynamic surface control (DSC) technique and designing novel adaptation laws. First, the 'explosion of terms' problem caused by backstepping method in the traditional adaptive robust control (ARC) is avoided. Meanwhile, through a new proof philosophy the asymptotical output tracking that the ARC possesses is theoretically preserved. Second, when persistent excitation (PE) condition satisfies, true parameter estimates could be acquired via designing CALs which integrate the information of estimation errors. Finally, simulation results are presented to illustrate the effectiveness of the proposed method.

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KEY WORDS: adaptive robust control; dynamic surface control; composite adaptation laws

1. INTRODUCTION

In the past decades, control of uncertain nonlinear systems with strict or semi-strict feedback forms has received much attention and various control algorithms have been considered [1–11]. Among them, an adaptive robust control (ARC) proposed by Bin Yao *et al.* claims that it could deal with the parametric and nonlinear uncertainties simultaneously [4, 5, 9, 12, 13]. The ARC combines deterministic robust control with adaptive control effectively and has been widely used in motor servomechanisms [14, 15], hard-disk control systems [16], hydraulic systems [17] and levitated systems [18]. Although ARC achieves many fruits in real application, it still has some drawbacks: *First*, due to using the backstepping design philosophy, its controller is quite complicated and tedious in implementation when the system order is equal to or greater than three. The control signal u will include the nth derivative of the first virtual control, the $(n-1)$th derivative of the second virtual control, and so on, which will lead to the 'explosion of terms' phenomena that can be seen in [4, 5, 9, 10]. In addition, the complicated and tedious control law will cost a lot of hardware resources and increase the unnecessary cost of control system in real application.

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Second, the parameter adaptation law in normal ARC is synthesized for a sole objective of reducing the output tracking error based on the Lyapunov stability analysis. This kind of gradient adaptation law only driven by tracking error signals has a simple form but often exhibits slow parameter convergence rate and rough estimate of parameter in comparison with other possible adaptive update laws (e.g. least square update law). That will cause the following problem: whether the parameters converge to their true value or not is unclear (i.e. the function of adaptive term in ARC hardly puts into great play).

To overcome the first drawback in ARC, a dynamic surface control (DSC) technique [19, 20] consists of a linear filter for derivative generation was proposed. It utilizes a simpler algebraic operation in lieu of the operation of differentiation. As a result, the tedious and complicated derivatives of virtual control laws are avoided. In [21], Zi-Jiang Yang *et al.* designed an ARC using the DSC technique for a levitated system for the first time. Subsequently, they expanded this method to a class of nonlinear systems in semi-strict feedback form [22]. Afterwards, Jie Chen research group inherits and develops this approach and applied it in servomechanisms successfully [23]. However, all the above methods could show that the signals in the system are bounded even if only parametric uncertainty exists and did not do any further contributions on the parameter convergence.

It is well known that parameter convergence is an old and important issue since it enhances the overall stability and robustness of the closed-loop adaptive control systems [24]. Variety of parameter convergence results have been developed to compensate for the linear-in-the-parameters uncertainty in nonlinear systems [1, 4–10, 25–29]. Most of this research has exploited the Lyapunov-based and estimation-based techniques. Lyapunov-based methods could always guarantee the stability of the closed-loop systems, result in a simple gradient type structure of update law but often lead to weakness parameter convergence as the update law is usually designed to cancel the cross terms. While estimation-based methods make the update law design freely where the modular design method [29] is always employed to guarantee the stability of systems with strong controllers. Therefore, many identification methods (e.g. least-square method and its varieties [30, 31]) can be adopted to guarantee well parameter convergence. For example, in order to solve the second drawback in normal ARC, Bin Yao *et al.* proposed an IARC method, in which the update law is estimation-based and the least-square method is used to achieve true parameter estimates [26]. Similarly, a modular ARC was investigated in [27] for a class of semi-strict feedback nonlinear systems where the Lyapunov-based update law is replaced by an estimation-based update law. However, these least-square methods and the modular design philosophy increase the complexity of controller design and controller's conservatism, respectively. This departs from our original intention. Thus, the research of this paper is motivated by the following question: *Can we retain the advantages of the original ARC* (*i.e. prescribed transient performance and asymptotical tracking performance in the presence of the parametric uncertainties only*) *when the DSC technique is involved and meanwhile seek a simpler update law from the Lyapunov stability analysis with fast and accurate parameter estimates?*

The results in this paper will give an assured answer. Herein, we propose an adaptive robust dynamic surface control (ARDSC) for a class of uncertain nonlinear system in semi-strict feedback based on composite adaptation laws. The property of the derivative of the virtual control $\dot{\alpha}_i$ defined in the next section is further analyzed and a composite gradient type of adaptive update law derived from the Lyapunov stability analysis is redesigned. As a result, we can theoretically prove that (1) not only all the signals in the system are uniformly ultimately bounded, (2) but also a faster asymptotical tracking is achieved even though the DSC technique is involved, (3) and true parameter estimates can be obtained when the persistent excitation condition is satisfied in the presence of parametric uncertainties only.

The remainder of the paper is organized as follows. In Section 2, the problem and some preliminaries are declared. Design procedures of controller and parameter adaptation are provided in Section 3. Stability and performance analysis of the system are given in Section 4. In Section 5 the advantages of the proposed ARDSC with composite adaptation law (CAL) are illustrated by some comparative simulations, and concluding remarks are shown in Section 6.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following semi-strict feedback uncertain nonlinear system

$$\begin{aligned}\dot{x}_i &= \varphi_i^{\mathrm{T}}(x_1,\ldots,x_i,t)\theta_i + b_i x_{i+1} + d_i(x,t), \quad i=1,\ldots,n-1,\\ \dot{x}_n &= \varphi_n^{\mathrm{T}}(x_1,\ldots,x_n,t)\theta_n + b_n u + d_n(x,t),\\ y &= x_1,\end{aligned} \tag{1}$$

where $x=[x_1,x_2,\ldots,x_n]^{\mathrm{T}}\in R^n$ is the state vector, $y\in R$ and $u\in R$ are the system output and input, respectively, b_i is the unknown input gain of the ith channel, $\theta_i\in R^{p_i}, i=1,\ldots,n$ represent the vectors of other unknown parameters, $d_i(x,t)$ is the uncertain nonlinearity in the ith channel and $\varphi_i(x_1,\ldots,x_i,t)\in R^{p_i}$, $i=1,\ldots,n$ are known smooth functions. As shown, system (1) is suffered from nonlinear uncertainties $d_i(x,t)$ and parametric uncertainties θ_i, b_i simultaneously. The problem is to design a bounded control law for the input u such that the system is stable and the output y tracks the desired output trajectory x_r as closely as possible in spite of the aforementioned uncertainties.

Throughout this paper, the following notations will be used. In general, the operation $\leqslant$ for two vectors is performed in terms of the corresponding elements of the vectors. Let $\hat{\star}$ denote the estimate of $\star$ (e.g. $\hat{\theta}_i$ for θ_i) and $\tilde{\star}$ is defined as $\tilde{\star}=\star-\hat{\star}$. $\|\star\|$ denotes the Euclidean norm of $\star$.

To facilitate control system design, the following assumptions and preliminaries are needed.

Assumption 1
The unknown parameter vectors $\theta_i\in\Omega_i (i=1,\ldots,n)$ and $\theta_b=[b_1,\ldots,b_n]^{\mathrm{T}}\in\Omega_b$, where $\Omega_i=\{\theta_i\in R^{p_i}|\theta_{i\min}\leqslant\theta_i\leqslant\theta_{i\max}\}$; $\Omega_b=\{\theta_b\in R^n|0<\theta_{b\min}\leqslant\theta_b\leqslant\theta_{b\max}\}$; $\theta_{i\min}$, $\theta_{i\max}$, $\theta_{b\min}$, $\theta_{b\max}$ are known vectors.

Assumption 2
The unknown uncertain nonlinearities $d_i(x,t)$ are assumed to be bounded by

$$|d_i(x,t)|\leqslant\delta_i, \quad i=1,\ldots,n, \tag{2}$$

where δ_i are known positive constants.

Assumption 3
The desired trajectory vectors are continuous and available, and $[x_r,\dot{x}_r,\ddot{x}_r]^{\mathrm{T}}\in\Omega_r$ with a known compact set $\Omega_r=\{[x_r,\dot{x}_r,\ddot{x}_r]^{\mathrm{T}}: x_r^2+\dot{x}_r^2+\ddot{x}_r^2\leqslant B_0\}\subset R^3$, whose size B_0 is a known positive constant.

Lemma 1 (*Krstic et al.* [*1*])
Consider the function $\phi: R_+\rightarrow R$. If $\phi, \dot{\phi}\in L_\infty$, and $\phi\in L_p$ for some $p\in[1,\infty)$, then

$$\lim_{t\rightarrow\infty}\phi(t)=0. \tag{3}$$

Fact 1
For $a\in R$, $b\in R$ and $c\in R_+$, the following inequality holds:

$$a\cdot b\leqslant ca^2+\frac{1}{4c}b^2. \tag{4}$$

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Discontinuous projection operator

In order to guarantee that the parameter estimates in the adaptive procedure are bounded, the discontinuous projection operator [4, 5, 22] is introduced herein with the following expression and properties:

$$\mathrm{Proj}_{\hat{\Theta}}(\bullet)=\begin{cases}0 & \text{if } \hat{\Theta}_i=\Theta_{i(\max)} \text{ and } \bullet>0,\\ 0 & \text{if } \hat{\Theta}_i=\Theta_{i(\min)} \text{ and } \bullet<0,\\ \bullet & \text{otherwise,}\end{cases} \tag{5}$$

where Θ, $\hat{\Theta}$ represent the unknown parameter vector to be online updated and its estimate respectively. The footnote i denotes the ith element. $i(\min)$, $i(\max)$ denote minimum and maximum values of the ith element of Θ, respectively. '$\bullet$' represents any reasonable adaptation function.

Choose the parametric adaptation as

$$\dot{\hat{\Theta}}=\mathrm{Proj}_{\hat{\Theta}}(\Gamma\pi), \tag{6}$$

where $\Gamma>0$ is a diagonal matrix, π is an adaptation function. It can be shown that for any adaptation function π, the projection mapping used in (6) guarantees

$$\text{Property 1: } \hat{\Theta}\in\Omega_{\Theta}=\{\hat{\Theta}:\Theta_{\min}\leqslant\hat{\Theta}\leqslant\Theta_{\max}\}, \tag{7a}$$

$$\text{Property 2: } \tilde{\Theta}^{\mathrm{T}}(\pi-\Gamma^{-1}\mathrm{Proj}_{\hat{\Theta}}(\Gamma\pi))\leqslant 0 \quad \forall\pi. \tag{7b}$$

3. DESIGN OF THE CONTROLLER AND PARAMETER ADAPTATION LAW

In this section, first we take the DSC technique to design an ARC instead of the integral backstepping method to avoid the so-called 'explosion of terms' problem. Then we elaborately design a novel update law to ensure the true parameter estimates.

3.1. Controller design

Unlike the traditional ARC designing procedure, a dynamic surface control approach is utilized in this paper. The ARDSC design is based on the following coordinates transformation: $s_i=x_i-x_{ir}$, $(i=1,\ldots,n)$, where x_{1r} equals to the desired trajectory x_r, and x_{ir} $(i>1)$ is the output of a first-order filter with input α_{i-1} which is denoted as a virtual control for the corresponding $(i-1)$th subsystem of system (1). At the last step, α_n is constructed as the system control law u.

The concrete design procedure is given as follows.

First, consider the ith $(i=1,\ldots,n-1)$ equation of system (1)

$$\dot{x}_i=\varphi_1^{\mathrm{T}}(x_1,\ldots,x_i)\theta_i+b_i x_{i+1}+d_i(x,t). \tag{8}$$

Define the ith dynamic surface as $s_i=x_i-x_{ir}$. Then its derivative is

$$\dot{s}_i=\varphi_i^{\mathrm{T}}(x_1,\ldots,x_i)\theta_i+b_i x_{i+1}+d_i(x,t)-\dot{x}_{ir}. \tag{9}$$

The purpose is to synthesize a virtual control function α_i for x_{i+1} so that the ith dynamic surface converges to zero or some small values with a guaranteed transient. Therefore the virtual control α_i is constructed as follows:

$$\begin{aligned}\alpha_i &= \alpha_{ia}+\alpha_{is},\\ \alpha_{ia} &= \frac{(-\varphi_i^{\mathrm{T}}(x_1,\ldots,x_i)\hat{\theta}_i+\dot{x}_{ir})}{\hat{b}_i},\\ \alpha_{is} &= \alpha_{is1}+\alpha_{is2}, \alpha_{is1}=-\frac{k_{is}}{b_{i\min}}s_i,\end{aligned} \tag{10}$$

where α_{ia} represents the adjustable model compensation; α_{is} is the robust control law consisting of two parts: α_{is1}, a simple proportional feedback in this case is used to stabilize the nominal system, and α_{is2} to be synthesized later represents robust feedback for attenuating the effect of model uncertainties.

Next define the $(i+1)$th dynamic surface $s_{i+1}=x_{i+1}-x_{(i+1)r}$ where $x_{(i+1)r}$ equals α_i passed through a first order low-pass filter, i.e.

$$\tau_{i+1}\dot{x}_{(i+1)r}+x_{(i+1)r}=\alpha_i, \quad x_{(i+1)r}(0)=\alpha_i(0). \tag{11}$$

Define $y_{i+1}=x_{(i+1)r}-\alpha_i$, $z_i=[s_i, y_{i+1}]^{\mathrm{T}}$ and $v_i=[s_{i+1}, \dot{\alpha}_i]^{\mathrm{T}}$, then the ith error equation $\mathscr{E}_i$ can be described as

$$\mathscr{E}_i:\ \dot{z}_i=A_i z_i+B_i v_i+D_i(x,t,\theta), \tag{12}$$

where

$$D_i(x,t,\theta)=\begin{bmatrix}\tilde{b}_i\alpha_{ia}+\varphi_i^{\mathrm{T}}(x_1,\ldots,x_i)\tilde{\theta}_i+d_i(x,t)+b_i\alpha_{is2}\\ 0\end{bmatrix}, \quad B_i=\begin{bmatrix}b_i & 0\\ 0 & -1\end{bmatrix},$$

$$A_i=\begin{bmatrix}-\dfrac{b_i k_{is}}{b_{i\min}} & b_i\\ 0 & -\dfrac{1}{\tau_{i+1}}\end{bmatrix},$$

v_i is considered as input.

Lemma 2

The error equation $\mathscr{E}_i$ is input-to-state stable (ISS) if the robust control α_{is2} satisfies the following conditions:

$$\begin{aligned}&\text{(i)}\ \ s_i[\tilde{b}_i\alpha_{ia}+\varphi_i^{\mathrm{T}}(x_1,\ldots,x_i)\tilde{\theta}_i+d_i(x,t)+b_i\alpha_{is2}]\leqslant\varepsilon_i,\\ &\text{(ii)}\ s_i\alpha_{is2}\leqslant 0,\end{aligned} \tag{13}$$

where ε_i is an arbitrary small positive constant.

Proof

Choose a positive-definite function (p. d. f.) $V_i=(1/2)z_i^{\mathrm{T}}z_i$. Noting (12), its time derivative is

$$\begin{aligned}\dot{V}_i &= z_i^{\mathrm{T}}A_i z_i+z_i^{\mathrm{T}}B_i v_i+z_i^{\mathrm{T}}D_i\\ &= -\frac{b_i k_{is}}{b_{i\min}}s_i^2+s_i[\tilde{b}_i\alpha_{ia}+\varphi_i^{\mathrm{T}}(x_i)\tilde{\theta}_i+d_i(x,t)+b_i\alpha_{is2}]+b_i s_i(y_{i+1}+s_{i+1})+y_{i+1}\left(-\frac{y_{i+1}}{\tau_{i+1}}-\dot{\alpha}_i\right).\end{aligned} \tag{14}$$

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From *Fact 1* and (13), the following inequality is obtained

$$\dot{V}_i \leqslant -k_{is}s_i^2+\varepsilon_i+2b_{i\max}s_i^2+\left(\frac{1}{4}b_{i\max}+1-\frac{1}{\tau_{i+1}}\right)y_{i+1}^2+\frac{1}{4}|\dot{\alpha}_i|^2+\frac{1}{4}b_{i\max}s_{i+1}^2. \tag{15}$$

Thus, there exists k_{is} and τ_{i+1} such that

$$\dot{V}_i \leqslant -\lambda_i V_i+\beta_i\|v_i\|^2+\varepsilon_i, \tag{16}$$

where λ_i and β_i are some positive constants. Then we can conclude that

$$\|z_i(t)\|^2 \leqslant \frac{\beta_i\|v_i(t)\|^2+\varepsilon_i}{\lambda_i}(1-\mathrm{e}^{-\lambda_i t})+\|z_i(0)\|^2\mathrm{e}^{-\lambda_i t}, \tag{17}$$

which indicates that $\mathscr{E}_i$ is ISS. □

Remark 1
According to the proof of Lemma 2, we know that as long as the disturbance-like term $D_i(x,t,\theta)$ is bounded, the input-to-state stability of error system $\mathscr{E}_i$ holds. Here, the purpose that we need the robust control term α_{is2} to satisfy the condition (13) is to achieve uniformly ultimately bounded results for the solutions of the whole closed-loop system and provide a guidance for designing the robust control term α_{is2}.

Finally, consider the nth equation of system (1) and define the nth dynamic surface $s_n = x_n - x_{nr}$. Then we have

$$\dot{s}_n = \varphi_n^{\mathrm{T}}(x_1,\ldots,x_n)\theta_n+b_n u+d_n(x,t)-\dot{x}_{nr}. \tag{18}$$

The overall control law $u=\alpha_n$ can be similarly constructed as

$$\begin{aligned}
\alpha_n &= \alpha_{na}+\alpha_{ns},\\
\alpha_{na} &= \frac{(-\varphi_n^{\mathrm{T}}(x_1,\ldots,x_n)\hat{\theta}_n+\dot{x}_{nr})}{\hat{b}_n},\\
\alpha_{ns} &= \alpha_{ns1}+\alpha_{ns2}, \alpha_{ns1}=-\frac{k_{ns}}{b_{n\min}}s_n.
\end{aligned} \tag{19}$$

For the sake of convenience, let $z_n = s_n$ and substitute (19) into (18), then the nth error equation $\mathscr{E}_n$ can be described as

$$\mathscr{E}_n:\ \dot{z}_n = -\frac{b_n k_{ns}}{b_{n\min}}z_n+\tilde{b}_n\alpha_{na}+\varphi_n^{\mathrm{T}}(x_1,\ldots,x_n)\tilde{\theta}_n+d_n(x,t)+b_n\alpha_{ns2}. \tag{20}$$

Lemma 3
The error equation $\mathscr{E}_n$ is stable, if α_{ns2} is chosen to satisfy the following two conditions:

$$\begin{aligned}
&\text{(i)}\ \ s_n[\tilde{b}_n\alpha_{na}+\varphi_n^{\mathrm{T}}(x_1,\ldots,x_n)\tilde{\theta}_n+d_n(x,t)+b_n\alpha_{ns2}]\leqslant\varepsilon_n,\\
&\text{(ii)}\ s_n\alpha_{ns2}\leqslant 0,
\end{aligned} \tag{21}$$

where ε_n is an arbitrary small positive constant.

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Proof

Choose a p. d. f. function $V_n = (1/2)z_n^2$. Its time derivative satisfies

$$\dot{V}_n = -\frac{b_n k_{ns}}{b_{n\min}} z_n^2 + s_n[\tilde{b}_n u_a + \varphi_n^{\mathrm{T}}(x_1, \ldots, x_n)\tilde{\theta}_n + d_n(x,t) + b_n u_{s2}]$$
$$\leqslant -\frac{b_n k_{ns}}{b_{n\min}} z_n^2 + \varepsilon_n. \tag{22}$$

It implies that there exists some control gain k_{ns} such that

$$\|z_n\|^2 \leqslant \frac{\varepsilon_n}{\lambda_n} + \left(\|z_n(0)\|^2 - \frac{\varepsilon_n}{\lambda_n}\right) \mathrm{e}^{-\lambda_n t}, \tag{23}$$

where the λ_n is a positive constant corresponding to k_{ns}. Thus the nth error equation $\mathscr{E}_n$ is stable. □

Remark 2

From the above design procedure, we know that the derivative of virtual control α_{n-1} is substituted by $\dot{x}_{nr}$. Thus the expression of final control law u is much simpler than those in [4, 9], which enhances the practicality of controller and develops the application fields of the ARC.

Remark 3

As shown in Section 3.1, the entire error system (closed-loop system) consists of $n-1$ ISS subsystems and one stable system. Thus the recursive system is bounded stable if the input vectors v_i, $i = 1, \ldots, n-1$ are bounded. In other words, from Lemma 2 we know that $s_{n-1}, \ldots, s_1$ will be also bounded.

3.2. Composite adaptation laws

In ARC, the adaptive update law derived from the Lyapunov stability analysis is usually driven by system tracking errors only, which leads to slow and rough parameter estimation. In this section, a novel update law is elaborately designed not losing the conciseness of the update law in normal ARC, whereas at the same time it overcomes the above problem.

Let $\Theta = [\theta_1^{\mathrm{T}}, \theta_2^{\mathrm{T}}, \ldots, \theta_n^{\mathrm{T}}, \theta_b^{\mathrm{T}}]^{\mathrm{T}} \in R^p$, where $p = p_1 + \cdots + p_n + n$, and rewrite system (1) to the following compact form

$$\dot{x} = F(x,u,t)\Theta + f(x,u,t) + d(x,t), \tag{24}$$

where $f(x,u,t) \in R^n$ is added for generality and represents the lumped effect of all known nonlinearities, which is zero for (1);

$$F(x,u) = \begin{bmatrix} \varphi_1^{\mathrm{T}} & \cdots & 0 & 0 & x_2 & 0 & \cdots & 0 \\ \mathbf{0} & \varphi_2^{\mathrm{T}} & \cdots & 0 & 0 & x_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \varphi_n^{\mathrm{T}} & 0 & 0 & \cdots & u \end{bmatrix}_{n\times p}, \quad d(x,t) = [d_1, \ldots, d_n]^{\mathrm{T}}.$$

Construct the following two filters:

$$\dot{\Xi} = -k(t)\Xi + F(x,u,t), \tag{25a}$$

$$\dot{\Xi}_0 = -k(t)(x + \Xi_0) - f(x,u,t), \tag{25b}$$

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where $k(t)=k_0+k_1(t), k_0>0, k_1(t)>0$. Let $\Upsilon=x+\Xi_0$. From (3.2), its time derivative is

$$\dot{\Upsilon}=F(x,u,t)\Theta-k(t)(x+\Xi_0)+d. \tag{26}$$

Define $\Phi=\Upsilon-\Xi\Theta$, then its derivative is

$$\dot{\Phi}=-k(t)\Phi+d, \tag{27}$$

which indicates that $\Phi\to 0$, when $t\to\infty$ on the condition of $d\equiv 0$.

Define $P(t)=\int_0^t \Xi^{\mathrm{T}}(\varsigma)\Xi(\varsigma)\,\mathrm{d}\varsigma$, $Q(t)=\int_0^t \Xi^{\mathrm{T}}(\varsigma)(\Upsilon(\varsigma)-\Phi(\varsigma))\,\mathrm{d}\varsigma$, we have $P(t)\Theta=Q(t)$. Let $\Pi=[s_1\varphi_1^{\mathrm{T}}(x_1),\ldots,s_n\varphi_n^{\mathrm{T}}(x),s_1\alpha_{1a},\ldots,s_n\alpha_{na}]^{\mathrm{T}}$. Then a novel adaptation law is chosen as

$$\dot{\hat{\Theta}}=\mathrm{Proj}(\Gamma(\Pi K(P\hat{\Theta}-Q))), \tag{28}$$

where $K>0$ is a diagonal matrix.

Remark 4

We call the adaptation law (28) as a CAL since, unlike References [4, 18], the adaptation law (28) makes full use of the parameter adaptation error $\tilde{\Theta}$ and tracking errors $s_i, i=1,\ldots,n$. In addition, it has the gradient type which is different from Reference [26]. As such, choosing a proper learning factor K and Γ, simple and accurate parameter estimation can be obtained.

Remark 5

From (28), we will obtain that $\dot{\tilde{\Theta}}=\mathrm{Proj}(-\Gamma s\phi(x,t)-\Gamma KP\tilde{\Theta})$. It means that the parameter convergence rate can be adjusted by learning factors Γ, K and faster convergence rate can be obtained through adjusting K when the controller parameters are the same. Furthermore, if the tracking error s converges to zero and the matrix ΓKP is positive definite, the parameter error $\tilde{\Theta}$ will converge to zero. While for the normal adaptation laws proposed in [4, 9], the parameter error dynamics is described as $\dot{\tilde{\Theta}}=\mathrm{Proj}(-\Gamma s\phi(x,t))$. This indicates that the parameter convergence rate depends on the tracking error whose convergence rate then depends on the controller parameters. Even more unfortunate, although the tracking error $s\to 0$, we have no idea whether the unknown parameters converge to their true values or not.

4. ANALYSIS OF THE STABILITY AND PERFORMANCE

From Lemma 3, we know that the nth dynamic surface s_n is uniformly bounded. Therefore s_{n-1} is bounded if $\dot{\alpha}_{n-1}$ is bounded. Keeping a job, a bounded tracking error s_1 can be deduced. The analysis in detail will be shown in Theorem 1.

Actually, the explicit term of $\dot{\alpha}_i$ can be expanded as follows:

$$\dot{\alpha}_i=\frac{1}{\hat{b}_i^2}\left\{\left[\left[-\frac{\partial\varphi_i^{\mathrm{T}}(x_1,\ldots,x_i)}{\partial(x_1,\ldots,x_i)}\begin{bmatrix}\dot{x}_1\\ \vdots\\ \dot{x}_i\end{bmatrix}\right]\hat{\theta}_i-\varphi_i^{\mathrm{T}}(x_1,\ldots,x_i)\dot{\hat{\theta}}_i+\ddot{x}_{ir}\right]\hat{b}_i-\varphi_i^{\mathrm{T}}(x_1,\ldots,x_i)\hat{\theta}_i\dot{\hat{b}}_i\right\}-\frac{k_{is}}{b_{i\min}}\dot{s}_i+\dot{\alpha}_{is2}. \tag{29}$$

In order to guarantee that the derivative of α_i is bounded by a continuous function $C_i\ (s_1,\ldots,s_{i+1},y_2,\ldots,y_{i+1},\hat{\theta}_1,\ldots,\hat{\theta}_i,\hat{b}_1,\ldots,\hat{b}_i,x_{1r},\dot{x}_{1r},\ddot{x}_{1r})$, i.e.

$$|\dot{\alpha}_i|\leqslant|C_i(s_1,\ldots,s_{i+1},y_2,\ldots,y_{i+1},\hat{\theta}_1,\ldots,\hat{\theta}_i,\hat{b}_1,\ldots,\hat{b}_i,x_{1r},\dot{x}_{1r},\ddot{x}_{1r})|, \tag{30}$$

where C_i is some continuous function with maximum value M_i on a compact set, the $\alpha_{is2}(i=1,\ldots,n)$ in this paper is chosen to be

$$\alpha_{is2} = -\frac{h_i}{2b_{i\min}\varepsilon_i}s_i, \tag{31}$$

where h_i is any constant satisfying $h_i \geqslant |b_{i\max}-b_{i\min}|^2|\alpha_{ia}|^2+\|\theta_{i\max}-\theta_{i\min}\|^2\|\varphi_i(x_1,\ldots,x_i)\|^2+\delta_i^2$.

Assumption 4
Given any positive constant ρ, all initial conditions of the subsystems $\mathscr{E}_i$ are in the compact set $\Omega_0 = \{(z_1,\ldots,z_n)|\sum_{i=1}^n \|z_i(0)\|^2 \leqslant 2\rho\}$.

Theorem 1
Consider the uncertain nonlinear system (1) with control law (19). If all the Assumptions 1–4 are established and the parameter adaptation is closed, then there exists k_{is1}, τ_{i+1} such that all the signals s_i, y_{i+1}, α_i are uniformly ultimately bounded and the steady-state tracking error s_1 is smaller than a prescribed error bound.

Proof
According to Lemmas 2 and 3, choose a Lyapunov function $V=\sum_{i=1}^n V_i$, then its time derivative satisfies

$$\dot{V} \leqslant \sum_{i=1}^{n}(-k_{is}s_i^2+\varepsilon_i)+\sum_{i=1}^{n-1}\left(2b_{i\max}s_i^2+\frac{1}{4}b_{i\max}s_{i+1}^2\right)+\sum_{i=1}^{n-1}\left[\left(-\frac{1}{\tau_{i+1}}+\frac{1}{4}b_{i\max}+1\right)y_{i+1}^2+\frac{1}{4}|\dot{\alpha}_i|^2\right]. \tag{32}$$

Thus, there exists k_{is1}, τ_{i+1} as follows:

$$\begin{cases} k_{1s} \geqslant 2b_{1\max}+\mu, \\ k_{is} \geqslant 2b_{i\max}+\dfrac{1}{4}b_{(i-1)\max}+\mu, \quad i=2,\ldots,n-1, \\ k_{ns} \geqslant \dfrac{1}{4}b_{n\max}+\mu, \\ \dfrac{1}{\tau_{j+1}} \geqslant \dfrac{1}{4}b_{j\max}+1+\mu, \quad j=1,\ldots,n-1, \end{cases} \tag{33}$$

such that

$$\dot{V} \leqslant -2\mu V+\upsilon, \tag{34}$$

where μ is some positive constant, $\upsilon=\sum_{i=1}^n \varepsilon_i+\sum_{i=1}^{n-1}\frac{1}{4}|\dot{\alpha}_i|^2$. From (30) and Assumption 4, υ is bounded. If we choose $\mu \geqslant \upsilon/2\rho$, then $\dot{V} \leqslant 0$. Thus, $V(t) \leqslant 2\rho$ is an invariant set on $t\in[0,\infty)$, i.e., (34) holds for all $t\in[0,\infty)$.

From (34), we have

$$0 \leqslant V(t) \leqslant \frac{\upsilon}{2\mu}+\left(V(0)-\frac{\upsilon}{2\mu}\right)e^{-2\mu t}. \tag{35}$$

It implies that s_i, y_{i+1} is uniformly ultimately bounded. Since Assumption 1 holds and parameter adaptation is closed, $\hat{\Theta}$ is bounded. Thus, from (30) the control law α_i is bounded. Moreover, the error bound is prescribed by choosing a proper constant μ. □

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Remark 6
Theorem 1 shows that the proposed ARDSC can achieve a guaranteed transient performance and final output tracking accuracy in general—in the sense that the exponentially converging rate 2μ and the bounds of the final tracking error index can be adjusted by suitably choosing the design parameters.

In the recent literatures with respect to ARDSC (e.g., References [21–23]), Theorem 1 is their ultimate conclusion even if only parametric uncertainties exist. Actually, the asymptotical tracking performance of ARDSC is intrinsic when the adaptive control is active. What leads to the conservative results is that only the bound of $\dot{\alpha}_i$ is simply considered. Therefore, in this paper, we further analyze the property of $\dot{\alpha}_i$ to expect a better result (e.g., asymptotical tracking theoretically) and show how to achieve true parameter estimates in the presence of the parametric uncertainties only, which is refined as Theorem 2.

Theorem 2
Consider the uncertain nonlinear system (1) while the control law is fixed as (19). The parameter adaptation law is given by (28). If all the Assumptions 1–4 are established and uncertain nonlinearities in system (1) are absent, i.e., $d_i(x,t)=0 (i=1,\ldots,n)$, then asymptotical output tracking and true parameter estimates can be achieved via design proper controller parameters k_{is}, τ_{i+1} and learning factors Γ, K.

Proof
Choose a Lyapunov function V_Θ as

$$V_\Theta = V + \frac{1}{2}\tilde{\Theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\Theta}. \tag{36}$$

According to the subsystems Σ_i and update law (28), its time derivative is

$$\begin{aligned}
\dot{V}_\Theta &= \sum_{i=1}^{n-1}\left[-\frac{b_i k_{is}}{b_{i\min}}s_i^2 + s_i\alpha_{ia}\tilde{b}_i + s_i\varphi_i^{\mathrm{T}}(x_1,\ldots,x_i)\tilde{\theta}_i + s_i(d_i(x,t)+b_i\alpha_{is2})\right]\\
&\quad + \sum_{i=1}^{n-1}\left[b_i s_i(y_{i+1}+s_{i+1}) + y_{i+1}\left(-\frac{y_{i+1}}{\tau_{i+1}} - \dot{\alpha}_i\right)\right] - \frac{b_n k_{ns}}{b_{n\min}}s_n^2 + s_n\tilde{b}_n u_a + s_n\varphi_n^{\mathrm{T}}(x)\tilde{\theta}_n + s_n(d_n(x,t)+b_n u_{s2})\\
&\quad + \tilde{\Theta}^{\mathrm{T}}\Gamma^{-1}\mathrm{Proj}(\Pi - K(P\hat{\Theta}-Q))\\
&= \sum_{i=1}^{n}\left[-\frac{b_i k_{is}}{b_{i\min}}s_i^2 + s_i(d_i(x,t)+b_i\alpha_{is2})\right] + \sum_{i=1}^{n-1}\left[b_i s_i(y_{i+1}+s_{i+1}) + y_{i+1}\left(-\frac{y_{i+1}}{\tau_{i+1}} - \dot{\alpha}_i\right)\right]\\
&\quad + \tilde{\Theta}^{\mathrm{T}}(\Pi - K(P\hat{\Theta}-Q)) + \tilde{\Theta}^{\mathrm{T}}\Gamma^{-1}Proj(\Pi - K(P\hat{\Theta}-Q)) + \tilde{\Theta}^{\mathrm{T}}\mathrm{K}(P\hat{\Theta}-Q).
\end{aligned} \tag{37}$$

If the uncertain nonlinearities $d_i(x,t)=0$ and the control parameters are chosen as (28), then the following inequality holds:

$$\dot{V}_\Theta \leqslant -2\mu V + \sum_{i=1}^{n-1}(-y_{i+1}\dot{\alpha}_i) + \tilde{\Theta}^{\mathrm{T}}K(P\hat{\Theta}-Q). \tag{38}$$

If the persistent exciting condition is satisfied, i.e. there exists a time instant t_c such that $P(t_c)=\int_0^{t_c}\Xi^{\mathrm{T}}(\varsigma)\Xi(\varsigma)\,\mathrm{d}\varsigma \geqslant \kappa I$, where κ is some positive constant, and I denotes the identity matrix with proper dimensions, then we have

$$\dot{V}_\Theta \leqslant -\mu z^{\mathrm{T}}z - \lambda_{\min}(KP)\tilde{\Theta}^{\mathrm{T}}\tilde{\Theta} + \sum_{i=1}^{n-1}(-y_{i+1}\dot{\alpha}_i), \tag{39}$$

where $\lambda_{\min}(KP)>0$ denotes the minimum eigenvalue of the matrix KP.

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Integrating both the sides of (39) on $t \in [0, \infty)$, the following inequality holds:

$$\int_0^{\infty} (\mu z^{\mathrm{T}} z + \lambda_{\min}(KP)\tilde{\Theta}^{\mathrm{T}}\tilde{\Theta})\,\mathrm{d}t \leqslant V_{\Theta}(0) - V_{\Theta}(t) + \sum_{i=1}^{n-1} \max_{0\leqslant t<\infty} |y_{i+1}||\alpha_i(t) - \alpha_i(0)|. \tag{40}$$

From the conclusion of Theorem 1, $V_{\Theta}(0) - V_{\Theta}(t)$ is bounded on $t \in [0, \infty)$ and $\sum_{i=1}^{n-1} \max_{0\leqslant t<\infty} |y_{i+1}||\alpha_i(t) - \alpha_i(0)|$ is also bounded. Thus, (40) implies that the signals $z, \tilde{\Theta} \in L_2[0, \infty)$. In addition, according to Theorem 1, z, $\tilde{\Theta} \in L_{\infty}[0, \infty)$. From the error equation $\mathscr{E}_i$ and parameter adaptation law (28), the signals derivatives $\dot{z}$, $\dot{\tilde{\Theta}} \in L_{\infty}[0, \infty)$. Therefore, according to Lemma 1, the signals z, $\tilde{\Theta} \to 0$ when $t \to 0$. □

Remark 7
The proper controller parameters k_{is}, τ_{i+1} mentioned in Theorem 2 first should satisfy (33) to guarantee the output tracking performance. We can require desired tracking performance by choosing larger μ, Γ carefully since the larger μ is selected, the smaller filter constant τ_{i+1} is needed and the larger control value will be. This makes the controller hard to implement in real systems. On the contrary, larger learning factors Γ, K will bring about a faster learning procedure but possibly make the learning oscillated especially when the projection operator is used. As a result, the proper learning factors also should be elaborately selected by trial-and-error method.

Remark 8
Theorem 2 indicates that by integrating the term of $\dot{\alpha}_i$ and using the composite update law (28), the asymptotical output tracking and parameter convergence can be proved. These results are much superior to those in [22, 23]. In addition, the update laws achieve the same results as those in [26] but with simpler forms.

5. COMPARATIVE SIMULATION RESULTS

To illustrate the advantages of the proposed control algorithm and CAL, some comparative simulation results are obtained from a third-order uncertain nonlinear system described by the following mathematical equations.

$$\begin{aligned}
\dot{x}_1 &= b_1 x_2 + \varphi_1^{\mathrm{T}}(x_1)\theta_1 + d_1(x,t),\\
\dot{x}_2 &= b_2 x_3 + \varphi_2^{\mathrm{T}}(x_1,x_2)\theta_2 + d_2(x,t),\\
\dot{x}_3 &= b_3 u + \varphi_3^{\mathrm{T}}(x_1,x_2,x_2)\theta_3 + d_3(x,t),\\
y &= x_1,
\end{aligned} \tag{41}$$

where

$$\begin{gathered}
\varphi_1(x_1) = x_1^2 \sin x_1, \quad d_1(x,t) = 0.3(x_1^2)^{1/3}\sin(x_2),\\
\varphi_2(x_1,x_2) = [x_1x_2, x_2\cos x_1]^{\mathrm{T}}, \quad d_2(x,t) = 0.3x_1x_2\sin x_3,\\
\varphi_3(x_1,x_2,x_3) = [x_1x_3, x_1^2\sin x_3, x_3\sin x_2]^{\mathrm{T}}, \quad d_3(x,t) = 0.3x_3\sin(10\pi t).
\end{gathered}$$

Suppose $\Theta = [\theta_1^{\mathrm{T}}, \theta_2^{\mathrm{T}}, \theta_3^{\mathrm{T}}, b_1, b_2, b_3]^{\mathrm{T}}$ and its nominal values are given as: $\Theta^0 = [2, 1, 1.5, 1, 2, 0.5, 1, 2, 3]^{\mathrm{T}}$. The bounds of unknown parameters are supposed as $\Theta_{\min} = [0, 0, 0, 0, 0, 0, 0.1, 0.2, 0.3]^{\mathrm{T}}$ and $\Theta_{\max} = [4, 2, 3, 2, 4, 1, 2, 4, 6]^{\mathrm{T}}$. The initial parameter estimates are $\hat{\Theta}(0) = [1, 0.5, 0.5, 0.2, 1, 0.1, 0.5, 0.5, 6]^{\mathrm{T}}$, which are different from the nominal values $\Theta^0 = [2, 1, 1.5, 1, 2, 0.5, 1, 2, 3]^{\mathrm{T}}$ to test the effect of parametric uncertainties. The objective is to design a control law u such that the output of the closed-loop system can track a reference input $x_r = \sin(2\pi t)$ as closely as possible. The controller parameters are chosen as follows: $\mu = 10$, $\alpha_{1s2} = -10s_1$,

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$\alpha_{2s2} = -10s_2$, $\alpha_{3s2} = -10s_3$, $\tau_2 = \tau_3 = 0.002$, $\Gamma = \text{diag}(50, 50, 50, 100, 100, 100, 100, 10, 1)$, $K = \text{diag}(5\times 10^4, 5\times 10^4, 5\times 10^4, 500, 1000, 500, 1\times 10^4, 1000, 1)$. When using the normal adaptation laws, K is set to zero.

In order to show the simplicity and effectiveness of the controller and adaptation law, the following three cases are elaborately designed.

5.1. *Case I*: *ARDSC with CAL and ARC considering uncertain nonlinearities*

The purpose in this case is to show that the ARDSC is much simpler than the ARC but retains the same advantages at least. The output tracking errors in this case are shown in Figure 1. Comparing the mathematical expressions of ARC in Section 3.2 of reference [26] with those in Section 3 of this paper, the simplicity of the ARDSC has been represented obviously. And from Figure 1, we can see that both the system tracking errors step into some boundaries with a prescribed transient stage and converge to small values gradually along with the parameter learning going on. Actually, the small values are corresponding to the bounds of uncertain nonlinearities $d_i(x,t)$. Moreover, when $d_i(x,t)$ vanish, these small values will converge to zero, namely, the tracking errors will converge to zero.

5.2. *Case II*: *ARDSC with CAL or normal adaptation laws(NAL) not considering uncertain nonlinearities*

In this case, we will show the effectiveness of CAL. The compared simulations are only executed in ARDSC between CAL and NAL. The results are shown in Figures 2 and 3.

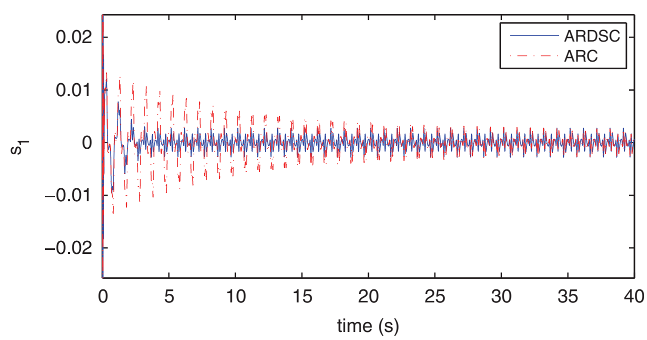

Figure 1. Output tracking errors using ARDSC and ARC, respectively.

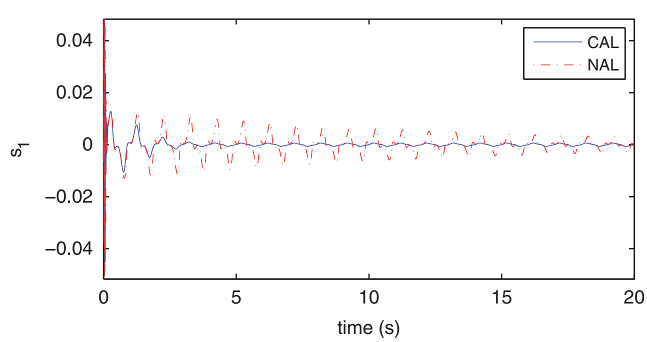

Figure 2. Output tracking errors not considering uncertain nonlinearities.

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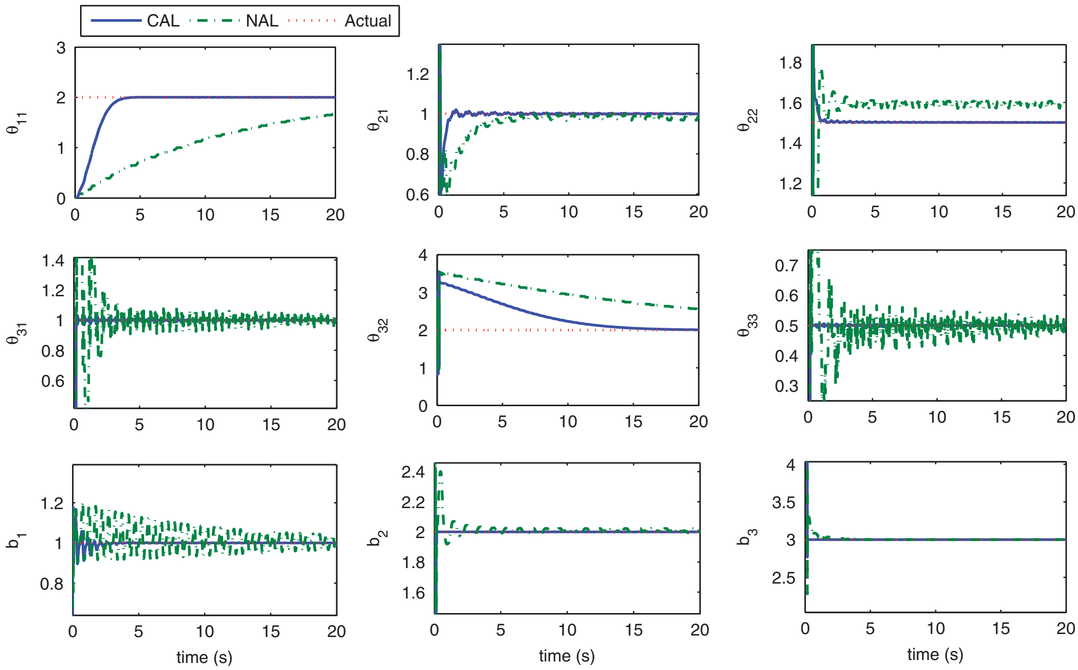

Figure 3. Parameter estimates not considering uncertain nonlinearities.

As we see in Figure 2, the tracking errors converge to zeros gradually. The difference between the curves is that the convergent rate of ARDSC with CAL is much faster than that with NAL, which indicates that utilizing the CAL is helpful to improve the tracking performance. In addition, Figure 3 shows that ARDSC with CAL could not only leads to a more accurate estimation of unknown parameters than that with NAL, but also a faster parameter convergence rate. All the results illustrated in Figures 2 and 3 imply that designing a novel adaptation law is a very useful way to improve the performance of control and parameter estimation.

5.3. Case III: ARDSC with CAL or NAL considering uncertain nonlinearities

The following simulation shows that even the system is affected by uncertain nonlinearities, ARDSC with CAL can still obtain well control performance and parameter estimation. The simulation results are shown in Figures 4 and 5. From Figures 4 and 5, we could see that both the tracking error and parameter estimations are with a faster convergence rate and higher precision because of using CAL. Thus, no matter the uncertain nonlinearities exist or not, the proposed ARDSC with CAL can achieve more satisfying tracking performance and parameter estimation.

6. CONCLUSIONS

This paper discusses an ARDSC for uncertain nonlinear systems in semi-strict feedback form. Two issues are commendably solved. One is that the ‘explosion of terms’ problem is overcome via introducing the DSC technique without losing the original advantages of ARC. Another is that parameter estimates can converge to their true

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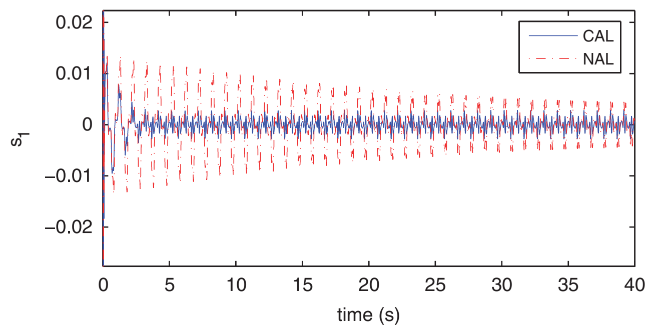

Figure 4. Output tracking errors considering uncertain nonlinearities.

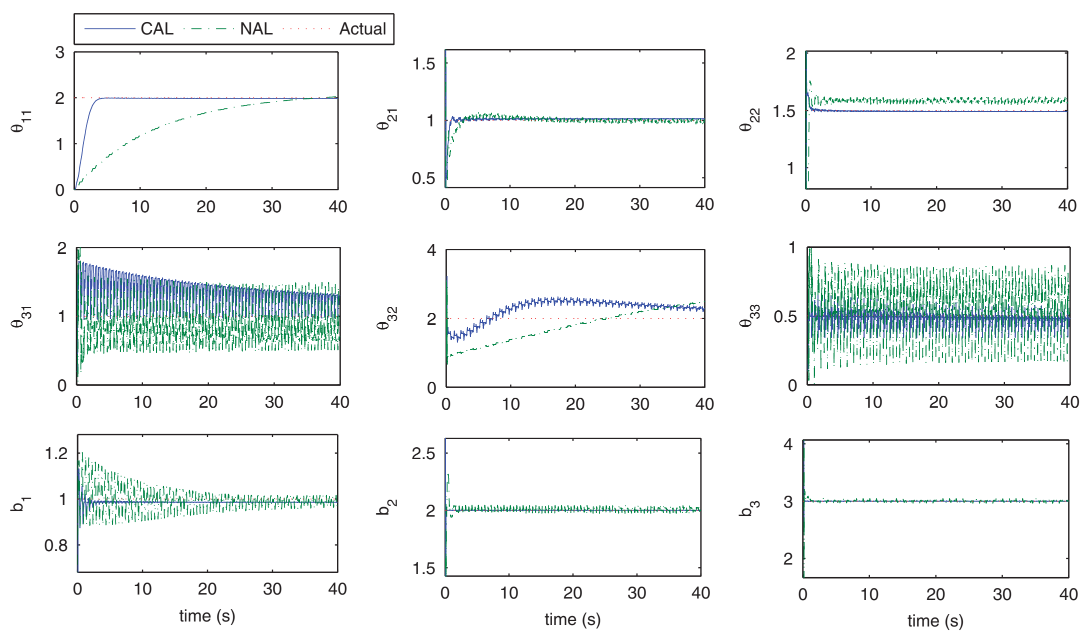

Figure 5. Parameter estimates considering uncertain nonlinearities.

values faster via designing composite update laws, which leads to a better tracking performance. Naturally, all the improvements claimed above are theoretically proved and verified by simulation results.

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