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Delay-dependent stability and stabilization of neutral time-delay systems

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SUMMARY

This paper is concerned with the problem of stability and stabilization of neutral time-delay systems. A new delay-dependent stability condition is derived in terms of linear matrix inequality by constructing a new Lyapunov functional and using some integral inequalities without introducing any free-weighting matrices. On the basis of the obtained stability condition, a stabilizing method is also proposed. Using an iterative algorithm, the state feedback controller can be obtained. Numerical examples illustrate that the proposed methods are effective and lead to less conservative results. Copyright © 2008 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Many practical systems, such as distributed networks containing lossless transmission lines [1] and population ecology [2], can be modeled by neutral time-delay systems. Therefore, the problem of the stability and stabilization of neutral time-delay systems has attracted considerable attention during the past few years. Using the Lyapunov–Razumikhin functional approach or the Lyapunov–Krasovskii functional approach, delay-independent [3, 4] and delay-dependent stability criteria [5–11] have been proposed. Since delay-independent conditions are usually more conservative

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than the delay-dependent conditions, more attention has been paid to the study of delay-dependent conditions. For example, a delay-dependent stability criterion for uncertain neutral systems with time-varying discrete delay was obtained in [12] based on a model transformation and Park's inequality [13]. A descriptor model transformation was introduced in [14–16] and stability conditions were developed for neutral time-delay systems. Based on the descriptor model transformation and the decomposition technique of discrete-delay term matrix, Han [17] put forward a stability test for neutral systems with time varying discrete and distributed delays. The stability of neutral systems with time-varying delay and its application to a partial element equivalent circuit model was considered in [18] based on the descriptor model transformation method. He *et al.* [19] developed a delay-dependent stability condition using the free-weighting matrices method, which did not use any model transformations or bounding techniques for cross terms. Moreover, this method combined with a parameterized model transformation method was used to derive a new delay-dependent stability criterion and a stabilizing method for uncertain neutral systems [20]. An augmented Lyapunov functional [21] was introduced to investigate the asymptotic stability of neutral time-delay systems. Two equivalent delay-dependent stability criteria were proposed. Recently, a novel augmented Lyapunov functional has been introduced in [22] to derive the robust stability of uncertain neutral systems. However, it can be found that the existing Lyapunov functional introduced in the literature only contains some integral terms, for example $\int_{t-\tau}^{t} x^{\mathrm{T}}(s)Qx(s)\,\mathrm{d}s$, and double-integral terms, for example $\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta$. If some triple-integral terms are introduced in the Lyapunov functional, what results can be obtained? This idea motivates this study.

In this paper, we introduce a new form of the Lyapunov functional that contains a triple-integral term $\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta$. Two integral inequalities are used to derive a new delay-dependent stability criterion without introducing any free-weighting matrices. Based on this criterion, a method of designing a stabilizing state feedback controller is also presented.

2. PROBLEM FORMULATION

Consider the following neutral time-delay system:

$$
\begin{aligned}
\dot{x}(t) - C\dot{x}(t-\tau) &= Ax(t) + A_1 x(t-\tau) + Bu(t), \quad t>0 \\
x(t) &= \phi(t), \quad t \in [-\tau, 0]
\end{aligned} \tag{1}
$$

where $x(t)\in\mathscr{R}^n$ is the state vector, $u(t)\in\mathscr{R}^m$ is the control input, $\tau>0$ is the constant delay. The initial condition $\phi(t)$ is a continuously differentiable vector-valued function, A, A_1, $C\in\mathscr{R}^{n\times n}$ and $B\in\mathscr{R}^{n\times m}$ are constant system matrices.

Throughout this paper, it is assumed that all the eigenvalues of C are inside the unit circle [23], which guarantees that the differential equation $\mathscr{D}x_t = x(t) - Cx(t-\tau) = 0$ is asymptotically stable for all τ.

The objective of this paper is to derive a less conservative delay-dependent stability condition and to design a state feedback controller $u(t) = Kx(t)$ to stabilize system (1).

Before moving on, the following lemma is introduced, which plays an important role in the development of the main results.

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Lemma 1
For any constant matrix $Z=Z^{\mathrm{T}}>0$ and a scalar $\tau>0$ such that the following integrations are well defined, then

(1)

$$-\int_{t-\tau}^{t}\varrho^{\mathrm{T}}(s)Z\varrho(s)\,\mathrm{d}s\leqslant-\frac{1}{\tau}\left(\int_{t-\tau}^{t}\varrho(s)\,\mathrm{d}s\right)^{\mathrm{T}}Z\left(\int_{t-\tau}^{t}\varrho(s)\,\mathrm{d}s\right)$$

(2)

$$-\int_{-\tau}^{0}\int_{t+\theta}^{t}\varrho^{\mathrm{T}}(s)Z\varrho(s)\,\mathrm{d}s\,\mathrm{d}\theta\leqslant-\frac{2}{\tau^2}\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}\varrho(s)\,\mathrm{d}s\,\mathrm{d}\theta\right)^{\mathrm{T}}Z\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}\varrho(s)\,\mathrm{d}s\,\mathrm{d}\theta\right)$$

Proof
Inequality (1) was proposed in [24]. For the proof of inequality (2), notice that

$$\begin{bmatrix}\varrho^{\mathrm{T}}(s)Z\varrho(s) & \varrho^{\mathrm{T}}(s)\\ \varrho(s) & Z^{-1}\end{bmatrix}\geqslant 0 \tag{2}$$

Integration of (2) from $t+\theta$ to t, where $-\tau\leqslant\theta\leqslant 0$, yields

$$\begin{bmatrix}\int_{t+\theta}^{t}\varrho^{\mathrm{T}}(s)Z\varrho(s)\,\mathrm{d}s & \int_{t+\theta}^{t}\varrho^{\mathrm{T}}(s)\,\mathrm{d}s\\ \int_{t+\theta}^{t}\varrho(s)\,\mathrm{d}s & -\theta Z^{-1}\end{bmatrix}\geqslant 0 \tag{3}$$

Integration of (3) from $-\tau$ to 0 yields

$$\begin{bmatrix}\int_{-\tau}^{0}\int_{t+\theta}^{t}\varrho^{\mathrm{T}}(s)Z\varrho(s)\,\mathrm{d}s\,\mathrm{d}\theta & \int_{-\tau}^{0}\int_{t+\theta}^{t}\varrho^{\mathrm{T}}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\ \int_{-\tau}^{0}\int_{t+\theta}^{t}\varrho(s)\,\mathrm{d}s\,\mathrm{d}\theta & -\int_{-\tau}^{0}\theta Z^{-1}\,\mathrm{d}\theta\end{bmatrix}\geqslant 0 \tag{4}$$

Equation (4) is equivalent to inequality (2) according to Schur complements. The proof has been completed. □

3. MAIN RESULTS

The following theorem presents a sufficient stability condition for system (1) with $u(t)=0$.

Theorem 1
Given a scalar $\tau>0$, system (1) with $u(t)=0$ is asymptotically stable if there exist

$$P=\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ * & P_{22} & P_{23}\\ * & * & P_{33}\end{bmatrix}>0,\quad Q=\begin{bmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{bmatrix}>0,\quad Z=\begin{bmatrix}Z_{11} & Z_{12}\\ * & Z_{22}\end{bmatrix}>0$$

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and $R>0$ with appropriate dimensions such that

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & A^{\mathrm{T}}Y \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & A_1^{\mathrm{T}}Y \\ * & * & \Omega_{33} & \Omega_{34} & C^{\mathrm{T}}Y \\ * & * & * & \Omega_{44} & 0 \\ * & * & * & * & -Y \end{bmatrix} < 0 \tag{5}$$

where

$$\begin{aligned}
\Omega_{11} &= P_{13}+P_{13}^{\mathrm{T}}+Q_{11}+\tau Z_{11}+P_{11}A+A^{\mathrm{T}}P_{11}+Q_{12}A+A^{\mathrm{T}}Q_{12}^{\mathrm{T}} \\
&\quad +\tau Z_{12}A+\tau A^{\mathrm{T}}Z_{12}^{\mathrm{T}}-\frac{1}{\tau}Z_{22}-2R \\
\Omega_{12} &= -P_{13}+P_{23}^{\mathrm{T}}+P_{11}A_1+A^{\mathrm{T}}P_{12}+Q_{12}A_1+\tau Z_{12}A_1+\frac{1}{\tau}Z_{22} \\
\Omega_{13} &= P_{12}+P_{11}C+Q_{12}C+\tau Z_{12}C \\
\Omega_{14} &= P_{33}+A^{\mathrm{T}}P_{13}-\frac{1}{\tau}Z_{12}^{\mathrm{T}}+\frac{2}{\tau}R \\
\Omega_{22} &= -P_{23}-P_{23}^{\mathrm{T}}-Q_{11}+P_{12}^{\mathrm{T}}A_1+A_1^{\mathrm{T}}P_{12}-\frac{1}{\tau}Z_{22} \\
\Omega_{23} &= P_{22}-Q_{12}+P_{12}^{\mathrm{T}}C \\
\Omega_{24} &= -P_{33}+A_1^{\mathrm{T}}P_{13}+\frac{1}{\tau}Z_{12}^{\mathrm{T}} \\
\Omega_{33} &= -Q_{22} \\
\Omega_{34} &= P_{23}+C^{\mathrm{T}}P_{13} \\
\Omega_{44} &= -\frac{1}{\tau}Z_{11}-\frac{2}{\tau^2}R \\
Y &= Q_{22}+\tau Z_{22}+\tfrac{1}{2}\tau^2 R
\end{aligned}$$

Proof
Choose a Lyapunov–Krasovskii functional candidate as

$$\begin{aligned}
V(t) &= \zeta^{\mathrm{T}}(t)P\zeta(t)+\int_{t-\tau}^{t}\varrho^{\mathrm{T}}(s)Q\varrho(s)\,\mathrm{d}s+\int_{-\tau}^{0}\int_{t+\theta}^{t}\varrho^{\mathrm{T}}(s)Z\varrho(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad +\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta
\end{aligned} \tag{6}$$

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where

$$\zeta^{\mathrm{T}}(t)=\left[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau)\left(\int_{t-\tau}^{t}x(s)\,\mathrm{d}s\right)^{\mathrm{T}}\right],\quad \varrho^{\mathrm{T}}(s)=[x^{\mathrm{T}}(s)\ \ \dot{x}^{\mathrm{T}}(s)]$$

Taking the time derivative of $V(t)$ along the trajectory of system (1) yields

$$\begin{aligned}\dot{V}(t)=&\,2\zeta^{\mathrm{T}}(t)P\dot{\zeta}(t)+\varrho^{\mathrm{T}}(t)Q\varrho(t)-\varrho^{\mathrm{T}}(t-\tau)Q\varrho(t-\tau)+\tau\varrho^{\mathrm{T}}(t)Z\varrho(t)\\&-\int_{t-\tau}^{t}\varrho^{\mathrm{T}}(s)Z\varrho(s)\,\mathrm{d}s+\frac{1}{2}\tau^{2}\dot{x}(t)^{\mathrm{T}}(t)R\dot{x}(t)-\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\end{aligned}\tag{7}$$

Using Lemma 1, one can obtain that

$$-\int_{t-\tau}^{t}\varrho^{\mathrm{T}}(s)Z\varrho(s)\,\mathrm{d}s\leqslant-\frac{1}{\tau}\left(\int_{t-\tau}^{t}\varrho(s)\,\mathrm{d}s\right)^{\mathrm{T}}Z\int_{t-\tau}^{t}\varrho(s)\,\mathrm{d}s\tag{8}$$

and

$$\begin{aligned}&-\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\\&\leqslant-\frac{2}{\tau^{2}}\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right)^{\mathrm{T}}R\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\theta\right)\\&=-\frac{2}{\tau^{2}}\left(\tau x(t)-\int_{t-\tau}^{t}x(s)\,\mathrm{d}s\right)^{\mathrm{T}}R\left(\tau x(t)-\int_{t-\tau}^{t}x(s)\,\mathrm{d}s\right)\end{aligned}\tag{9}$$

Substituting (8)–(9) into (7) yields

$$\dot{V}(t)\leqslant\xi^{\mathrm{T}}(t)[\Omega+A_c^{\mathrm{T}}YA_c]\xi(t)\tag{10}$$

where

$$\Omega=\begin{bmatrix}\Omega_{11}&\Omega_{12}&\Omega_{13}&\Omega_{14}\\ *&\Omega_{22}&\Omega_{23}&\Omega_{24}\\ *&*&\Omega_{33}&\Omega_{34}\\ *&*&*&\Omega_{44}\end{bmatrix}$$

$$A_c=[A\ \ A_1\ \ C\ \ 0]$$

$$\xi^{\mathrm{T}}(t)=\left[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-\tau)\ \ \dot{x}^{\mathrm{T}}(t-\tau)\ \ \left(\int_{t-\tau}^{t}x(s)\,\mathrm{d}s\right)^{\mathrm{T}}\right]$$

By Schur complement, $\Omega+A_c^{\mathrm{T}}YA_c<0$ is equivalent to (5), which implies $\dot{V}(t)<0$. Hence, system (1) is asymptotically stable. □

Remark 1

By constructing a new augmented Lyapunov functional, a new delay-dependent stability criterion is obtained in Theorem 1. The proposed augmented Lyapunov functional is more general than those in [19, 20, 25]. Compared with the Lyapunov functional in [21], our Lyapunov functional contains an additional triple-integral term $\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,\mathrm{d}s\,\mathrm{d}\lambda\,\mathrm{d}\theta$, which plays a key role in the further reduction of conservativeness. To the best of the authors' knowledge, such a type of Lyapunov functional is used for the first time to derive a stability criterion for neutral time-delay systems.

Remark 2

If setting $R=\varepsilon I$ with ε being a sufficiently small positive scalar in Theorem 1, a corollary can be directly obtained (for limitations of space, this corollary is omitted here). Furthermore, following the similar line as in [26], it can be proved that this corollary is equivalent to Theorem 1 and 2 in [21], where the improvements over [16, 20] are demonstrated. Thus, results in [21] can be covered by Theorem 1 in this paper. This also proves theoretically that Theorem 1 is less conservative than results in [21]. Furthermore, two integral inequalities are used to derive Theorem 1 and no additional free-weighting matrices are introduced in the derivation except for Lyapunov matrices. Thus, the method proposed in this paper may have less decision variables than the well-known free-weighting matrix method [19–21].

On the basis of Theorem 1, the problem of stabilization of system (1) is considered. This can be concluded in the following theorem:

Theorem 2

Given a scalar $\tau>0$, system (1) with the memoryless state feedback controller $u(t)=\bar{K}X^{-1}x(t)$ is asymptotically stable if there exist

$$\hat{P}=\begin{bmatrix} X & \hat{P}_{12} & \hat{P}_{13} \\ * & \hat{P}_{22} & \hat{P}_{23} \\ * & * & \hat{P}_{33} \end{bmatrix}>0, \quad \hat{Q}=\begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} \\ * & \hat{Q}_{22} \end{bmatrix}>0, \quad \hat{Z}=\begin{bmatrix} \hat{Z}_{11} & \hat{Z}_{12} \\ * & \hat{Z}_{22} \end{bmatrix}>0, \quad \hat{R}>0$$

$J>0$ and $\bar{K}$ with appropriate dimensions such that

$$\begin{bmatrix} \hat{\Omega} & 0 & \Lambda & \Upsilon \\ * & -\hat{Y} & \hat{Y} & 0 \\ * & * & -J & 0 \\ * & * & * & -XJ^{-1}X \end{bmatrix}<0 \tag{11}$$

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where

$$\hat{\Omega}=\begin{bmatrix}\hat{\Omega}_{11} & \hat{\Omega}_{12} & \hat{\Omega}_{13} & \hat{\Omega}_{14}\\ * & \hat{\Omega}_{22} & \hat{\Omega}_{23} & \hat{\Omega}_{24}\\ * & * & \hat{\Omega}_{33} & \hat{\Omega}_{34}\\ * & * & * & \hat{\Omega}_{44}\end{bmatrix}$$

$$\Lambda=[\hat{Q}_{12}^{\mathrm{T}}+\tau\hat{Z}_{12}^{\mathrm{T}}\ \ \hat{P}_{12}\ \ 0\ \ \hat{P}_{13}]^{\mathrm{T}}$$

$$\Upsilon=[AX+B\bar{K}\ \ A_1X\ \ CX\ \ 0]^{\mathrm{T}}$$

$$\hat{\Omega}_{11}=\hat{P}_{13}+\hat{P}_{13}^{\mathrm{T}}+\hat{Q}_{11}+\tau\hat{Z}_{11}+AX+B\bar{K}+XA^{\mathrm{T}}+\bar{K}^{\mathrm{T}}B^{\mathrm{T}}-\frac{1}{\tau}\hat{Z}_{22}-2\hat{R}$$

$$\hat{\Omega}_{12}=-\hat{P}_{13}+\hat{P}_{23}^{\mathrm{T}}+A_1X+\frac{1}{\tau}\hat{Z}_{22}$$

$$\hat{\Omega}_{13}=\hat{P}_{12}+CX$$

$$\hat{\Omega}_{14}=\hat{P}_{33}-\frac{1}{\tau}\hat{Z}_{12}^{\mathrm{T}}+\frac{2}{\tau}\hat{R}$$

$$\hat{\Omega}_{22}=-\hat{P}_{23}-\hat{P}_{23}^{\mathrm{T}}-\hat{Q}_{11}-\frac{1}{\tau}\hat{Z}_{22}$$

$$\hat{\Omega}_{23}=\hat{P}_{22}-\hat{Q}_{12}$$

$$\hat{\Omega}_{24}=-\hat{P}_{33}+\frac{1}{\tau}\hat{Z}_{12}^{\mathrm{T}}$$

$$\hat{\Omega}_{33}=-\hat{Q}_{22}$$

$$\hat{\Omega}_{34}=\hat{P}_{23}$$

$$\hat{\Omega}_{44}=-\frac{1}{\tau}\hat{Z}_{11}-\frac{1}{\tau^2}\hat{R}$$

$$\hat{Y}=\hat{Q}_{22}+\tau\hat{Z}_{22}+\tfrac{1}{2}\tau^2\hat{R}$$

Proof

Applying the controller $u(t)=Kx(t)$ into system (1) yields

$$\dot{x}(t)-C\dot{x}(t-\tau)=(A+BK)x(t)+A_1x(t-\tau) \tag{12}$$

Following a similar method in [25], replacing A in (5) with $A+BK$, pre- and post-multiplying both sides of (5) with diag$\{X,X,X,X,X\}$ and its transpose, where $X=P_{11}^{-1}$, and defining $X(\cdot)X=\hat{(\cdot)}$ and $\bar{K}=KX$ yield

$$\Xi+\bar{\Lambda}X^{-1}\bar{\Upsilon}^{\mathrm{T}}+\bar{\Upsilon}X^{-1}\bar{\Lambda}^{\mathrm{T}}<0 \tag{13}$$

where

$$\Xi = \begin{bmatrix} \hat{\Omega} & 0 \\ * & -\hat{Y} \end{bmatrix}$$

$$\bar{\Lambda} = [\hat{Q}_{12}^{\mathrm{T}} + \tau \hat{Z}_{12}^{\mathrm{T}} \ \ \hat{P}_{12} \ \ 0 \ \ \hat{P}_{13} \ \ \hat{Y}]^{\mathrm{T}}$$

$$\bar{\Upsilon} = [AX + B\bar{K} \ \ A_1 X \ \ CX \ \ 0 \ \ 0]^{\mathrm{T}}$$

Clearly, the following inequality holds for any $J>0$ as:

$$\bar{\Lambda} X^{-1} \bar{\Upsilon}^{\mathrm{T}} + \bar{\Upsilon} X^{-1} \bar{\Lambda}^{\mathrm{T}} \leqslant \bar{\Lambda} J^{-1} \bar{\Lambda}^{\mathrm{T}} + \bar{\Upsilon} X^{-1} J X^{-1} \bar{\Upsilon}^{\mathrm{T}} \tag{14}$$

Substituting (14) into (13) and applying Schur complement yields (11). □

It should be noted that (11) is not a linear matrix inequality (LMI) because of the nonlinear term $XJ^{-1}X$. A simple way to solve it is to set $J=\lambda X$, where $\lambda>0$ is a tuning parameter. However, this method may increase the conservativeness. Similar to [25, 27, 28], an iterative algorithm is applied to obtain a suboptimal solution. First, a new variable $L>0$ is introduced such that $XJ^{-1}X \geqslant L$, which is equivalent to $X^{-1}JX^{-1} \leqslant L^{-1}$. Letting $H=L^{-1}$, $M=X^{-1}$, $F=J^{-1}$ and following a similar method in [25, 27, 28], the problem of finding a feasible solution of non-convex condition (11) can be converted to a minimization problem involving LMI conditions:

Minimize Trace $(LH+XM+JF)$

Subject to

$$\begin{bmatrix} H & M \\ M & F \end{bmatrix} \geqslant 0, \quad \begin{bmatrix} L & I \\ I & H \end{bmatrix} \geqslant 0, \quad \begin{bmatrix} X & I \\ I & M \end{bmatrix} \geqslant 0, \quad \begin{bmatrix} J & I \\ I & F \end{bmatrix} \geqslant 0 \tag{15}$$

$$\begin{bmatrix} \hat{\Omega} & 0 & \Lambda & \Upsilon \\ * & -\hat{Y} & \hat{Y} & 0 \\ * & * & -J & 0 \\ * & * & * & -L \end{bmatrix} < 0 \tag{16}$$

The above minimization problem can be solved using some algorithms such as the cone complementarity algorithm [29] and the SLPMM algorithm [30]. In this paper, the more widely used cone complementarity algorithm is adopted.

Algorithm

1. Find a feasible solution $\{\bar{K}_0, \hat{P}_0, \hat{Q}_0, \hat{Z}_0, \hat{R}_0, J_0, F_0, X_0, M_0, L_0, H_0\}$ for the LMI (15) and (16). Set $k=0$;
2. Solve the following LMI optimization problem for the variables $\{\bar{K}, \hat{P}, \hat{Q}, \hat{Z}, \hat{R}, J, F, X, M, L, H\}$:
   Minimize Trace $(L_k H + H_k L + X_k M + M_k X + J_k F + F_k J)$
   Subject to (15) and (16)
   Set $L_{k+1}=L$, $H_{k+1}=H$, $X_{k+1}=X$, $M_{k+1}=M$, $J_{k+1}=J$, $F_{k+1}=F$;

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3. If condition (11) is satisfied, then variables obtained in Step 2 are feasible solutions and exit. If condition (11) is not satisfied within a specified maximum number of iterations, then exit, too. Otherwise, set $k=k+1$, and go to Step 2.

4. NUMERICAL EXAMPLES

In this section, some examples are given to show that the proposed results are improvements over the existing ones.

Example 1
Consider the following neutral time-delay system with:

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\quad A_1=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\quad C=\begin{bmatrix}c & 0\\ 0 & c\end{bmatrix},\quad 0\leqslant c<1$$

Table I lists the maximum upper bounds on the delay in case of different c's compared with those in the literature [16, 20–22, 31]. It is seen from Table I that the stability criterion proposed in this paper gives much less conservative results than those in the existing literature.

Remark 3
Recently, a delay discretization scheme has been proposed in [11]. This scheme is very effective in the reduction of the conservatism. Combine the Lyapunov functional proposed in this paper with the delay discretization scheme, and a further less conservative result can be obtained. Numerical examples illustrate that the obtained result is less conservative than those in [11]. Especially, with a discretization of the delay in two intervals, the upper bound on the delay when $c=0$ obtained in [11] is 5.71, while our result is 5.94. The less conservativeness of our result mainly owes to the introduction of the triple-integral term in the Lyapunov functional.

Example 2
Consider the following neutral time-delay system with:

$$A=\begin{bmatrix}-1.7073 & 0.6856\\ 0.2279 & -0.6368\end{bmatrix},\quad A_1=\begin{bmatrix}-2.5026 & -1.0540\\ -0.1856 & -1.5715\end{bmatrix}$$

$$C=\begin{bmatrix}0.0558 & 0.0360\\ 0.2747 & -0.1084\end{bmatrix}$$

Table I. Numerical results for Example 1.

c	0	0.1	0.3	0.5	0.7	0.9
Fridman and Shaked [16]	4.47	3.49	2.06	1.14	0.54	0.13
Han [31]	4.35	4.33	4.10	3.62	2.73	0.99
Wu *et al*. [20]	4.47	4.35	4.13	3.67	2.87	1.41
He *et al*. [21]	4.47	4.42	4.17	3.69	2.87	1.41
Parlakçi [22]	4.63	4.57	4.29	3.75	2.88	1.41
Theorem 1	5.30	5.21	4.85	4.20	3.19	1.49

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Table II. Numerical results for Example 3.

Methods	τ	K	No. of iteration
Li and de Souza [34]	0.6779	$[-0.1155\ \ -1.9839]$	—
Fridman and Shaked [35]	1.51	$[-58.31\ \ -294.935]$	—
Parlakçi [25]	8	$[-65.4058\ \ -76.7778]$	111
Our results	9	$[-44.1358\ \ -49.0181]$	94
	10	$[-86.3203\ \ -93.8552]$	164
	11	$[-153.1753\ \ -164.7362]$	247

The maximum upper bounds on the delay obtained in [8, 19–21] are 0.5735, 0.5937, 0.6054 and 0.6189, respectively. Using Theorem 1, the obtained value is 0.6612, which is much larger than those in [8, 19–21]. If setting $C=0$, this system reduces to a retarded type time-delay system. The upper bounds on the delay obtained in [16, 32, 33] are all 0.6903 and 0.7163 in [20], and 0.7918 in [21]. Using Theorem 1, the obtained value is 0.8418. Obviously, our criterion can lead to much less conservative results.

Example 3
Consider the following time-delay system with

$$A=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix},\quad A_1=\begin{bmatrix}-1 & -1\\ 0 & -0.9\end{bmatrix},\quad C=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\quad B=\begin{bmatrix}0\\ 1\end{bmatrix}$$

The proposed iterative algorithm was implemented on an Intel Core (TM) 2 Duo® processor at 2.20 GHz using Matlab LMI toolbox. Computation times for $\tau=9$, 10 and 11 are 109.35 s, 190.56 s and 288.30 s, respectively. The compared delay bounds and the feedback controller gains are listed in Table II. From Table II, it can be found that the proposed stabilization criterion provides a larger bound on the delay than those achieved in [25, 34, 35].

5. CONCLUSIONS

In this study, the problems of stability and stabilization of neutral time-delay systems have been investigated. A new delay-dependent stability criterion has been proposed. Owing to the new structure of the proposed Lyapunov functional, the obtained stability criterion is less conservative than the existing ones. On the basis of the obtained stability criterion, a stabilizing method is also presented. Numerical examples have shown the effectiveness of the proposed method.

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