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Delay-Dependent Robust H_∞ Filter Design for Uncertain Linear Systems with Time-Varying Delay

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Abstract A new delay-dependent robust H_∞ filtering design for uncertain linear systems with time-varying delay is investigated. Two kinds of time-varying delays are considered. One is differentiable uniformly bounded with a bounded delay derivative; the other is continuous uniformly bounded. A full-order filter is designed which ensures the asymptotic stability of the filtering error system and a prescribed level of H_∞ performance for all possible parameters which reside in a given polytope. By constructing a new Lyapunov functional which contains a triple integral term, new delay-dependent conditions for the existence of the H_∞ filter are derived which are less conservative than the existing ones. The filter gain can be obtained by solving a set of linear matrix inequalities (LMIs). Finally, two numerical examples are given to show the effectiveness and the advantages of the proposed method.

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1 Introduction

In the past few decades, the problem of state estimation has received much attention. Among these approaches, Kalman filtering is the most popular and is widely applied in many practical systems. However, this approach is no longer applicable when the knowledge of the statistics of the external disturbance is not exactly known. To overcome this drawback, an alternative H_∞ filtering approach was proposed in [2, 16]. On the other hand, the time delay is often encountered in many practical systems such as chemical process systems and networked control systems [12, 13, 25]. And, it is well known that time delay is always one of the main causes of poor control performance and instability. So, much attention has been devoted to the study of the design problem of H_∞ filtering for time-delay systems [1, 3–6, 9–11, 15, 17, 18, 21, 22, 24, 26]. Some delay-independent results [1, 17, 18] and delay-dependent results [5, 6, 10, 11, 15, 21, 22, 24] have been obtained. Since delay-dependent results are usually less conservative than delay-independent ones, much attention has been paid to the study of delay-dependent H_∞ filtering for time-delay systems. Recently, using descriptor model transformation combined with Park's inequalities, some delay-dependent filtering results have been obtained in [3, 4]. On the basis of a new integral inequality, robust H_∞ filtering for uncertain linear systems with time-varying delay has been investigated in [26] and numerical examples have illustrated that the obtained results are less conservative than those in [4, 6]. However, such results as in [26] are still conservative. The chosen Lyapunov functional in [26] is commonly used in many publications. An integral term, $\int_{t-\tau(t)}^{t} x^{\mathrm{T}}(s)E^{\mathrm{T}}QEx(s)\,ds$, and a double integral term $\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}RE\dot{x}(s)\,ds\,d\theta$ are contained in it. It does not contain any triple integral terms. It should be noted that Lyapunov functionals containing triple integral terms have been introduced to derive stability conditions for systems with discrete and distributed delays [14]. If a type of Lyapunov functional containing a triple integral term is used to study the H_∞ filtering design problem for linear systems with only discrete delays, then the following questions can be asked: what results can be obtained and does the introduction of the triple integral term still lead to less conservative results? These questions motivate this study.

In this paper, new delay-dependent existence conditions for the H_∞ filter are established by constructing a new Lyapunov functional. This is done by considering some useful terms when estimating the upper bound on the derivative of the Lyapunov functional and using the free-weighting matrices method. The obtained conditions are less conservative than the existing ones. Two kinds of time-varying delays are considered in this study. One is differentiable uniformly bounded with a bounded delay derivative; the other is continuous uniformly bounded. Finally, two numerical examples are given to demonstrate the effectiveness of the proposed methods.

2 Problem Formulation

Consider the following linear system with time-varying delay:

$$\begin{aligned}\dot{x}(t) &= A_0 x(t) + A_1 x\big(t-d(t)\big) + B\omega(t)\\ y(t) &= C_0 x(t) + C_1 x\big(t-d(t)\big) + D\omega(t)\\ z(t) &= L_0 x(t) + L_1 x\big(t-d(t)\big) + G\omega(t)\\ x(t) &= \phi(t), \quad t \in [-h, 0]\end{aligned} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $y(t) \in \mathcal{R}^m$ is the measurement vector, $z(t) \in \mathcal{R}^p$ is the signal to be estimated, $\omega(t) \in \mathcal{R}^l$ is the external disturbance signal, $\phi(t)$ is a continuously differentiable initial vector function, and A_0, A_1, B, C_0, C_1, D, L_0, L_1 and G are constant matrices with appropriate dimensions. The system matrices are partially known parameters. We denote

$$\chi := [A_0, A_1, B, C_0, C_1, D, L_0, L_1, G] \in \Sigma$$

where Σ is a given convex polyhedral domain, namely,

$$\Sigma := \left\{ \chi(\lambda) = \sum_{i=1}^{q} \lambda_i \chi_i ; 0 \le \lambda_i \le 1, \sum_{i=1}^{q} \lambda_i = 1 \right\} \tag{2}$$

where the q vertices of the polytope are described by

$$\chi_i = \big[A_0^{(i)}, A_1^{(i)}, B^{(i)}, C_0^{(i)}, C_1^{(i)}, D^{(i)}, L_0^{(i)}, L_1^{(i)}, G^{(i)}\big]$$

It is assumed that the time-varying delay $d(t)$ considered in this study satisfies the following two cases:

Case 1: $d(t)$ is a differentiable function satisfying

$$0 \le d(t) \le h, \qquad \dot{d}(t) \le \mu \tag{3}$$

Case 2: $d(t)$ is a continuous function satisfying

$$0 \le d(t) \le h \tag{4}$$

where h and μ are constants.

Consider the following full-order filter:

$$\begin{aligned}\dot{\hat{x}}(t) &= A_f \hat{x}(t) + B_f y(t), \quad \hat{x}(0) = 0\\ \hat{z}(t) &= C_f \hat{x}(t) + D_f y(t)\end{aligned} \tag{5}$$

where A_f, B_f, C_f, and D_f are filter parameters with appropriate dimensions to be determined.

By defining an augmented state vector $\eta(t) = [x^{\mathrm{T}}(t)\ \hat{x}^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $z_e(t) = z(t) - \hat{z}(t)$, the following augmented system can be obtained:

$$
\begin{aligned}
\hat{\eta}(t) &= \hat{A}_0\eta(t) + \hat{A}_1\eta\big(t-d(t)\big) + \hat{B}\omega(t) \\
z_e(t) &= \hat{C}_0\eta(t) + \hat{C}_1\eta\big(t-d(t)\big) + \hat{D}\omega(t) \\
\eta(t) &= \big[\phi^{\mathrm{T}}(t)\ 0\big]^{\mathrm{T}}, \quad t \in [-h, 0]
\end{aligned}
\tag{6}
$$

where

$$
\hat{A}_0 = \begin{bmatrix} A_0 & 0 \\ B_f C_0 & A_f \end{bmatrix}, \qquad \hat{A}_1 = \begin{bmatrix} A_1 & 0 \\ B_f C_1 & 0 \end{bmatrix}, \qquad \hat{B} = \begin{bmatrix} B \\ B_f D \end{bmatrix}
$$

$$
\hat{C}_0 = \begin{bmatrix} L_0 - D_f C_0 & -C_f \end{bmatrix}, \qquad \hat{C}_1 = \begin{bmatrix} L_1 - D_f C_1 & 0 \end{bmatrix}, \qquad \hat{D} = G - D_f D
$$

On the basis of the above discussion, the H_∞ filtering problem considered in this paper can be defined as follows.

H_∞ filtering problem: Given a scalar $\gamma > 0$, design a full-order filter of the form (5) such that the augmented error system (6) with $\omega(t) = 0$ is asymptotically stable for all the possible parameters lying in the given polytope for all the delays satisfying Case 1 or Case 2 and such that the H_∞ performance $\|z_e(t)\|_2 < \gamma\|\omega(t)\|_2$ is guaranteed for all nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$ for a prescribed $\gamma > 0$ under zero initial conditions.

3 Main Results

In this section, some new delay-dependent conditions for the existence of the H_∞ filter are derived. The design method of the H_∞ filter is also presented.

3.1 H_∞ Performance Analysis

For Case 1, the following theorem gives a sufficient condition for the existence of the H_∞ filter of the form (5).

Theorem 1 *For given scalars $h > 0$, $\gamma > 0$ and μ, the augmented system (6) is asymptotically stable with the H_∞ performance $\|z_e(t)\|_2 < \gamma\|\omega(t)\|_2$ for all the delays satisfying (3) and for all the parameters that belong to the uncertain polytope (2), if there exist some matrices*

$$
P = \begin{bmatrix} P_1 & P_2 & P_4 \\ \star & P_3 & P_5 \\ \star & \star & P_6 \end{bmatrix} > 0, \qquad Q_1 > 0, \qquad Q_2 > 0, \qquad Z_1 > 0
$$

$$
Z_2 > 0, \qquad R > 0, \qquad X = [X_{ij}]_{5\times 5} \geq 0
$$

$$Y=\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix}, \qquad M=\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{bmatrix}, \qquad H=\begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix}$$

with appropriate dimensions such that

$$\Phi^{(i)}=\begin{bmatrix} \Xi^{(i)}+hX & \Gamma_1^{(i)\mathrm{T}} & hA_c^{(i)\mathrm{T}}Z_2 & \frac{h^2}{2}A_c^{(i)\mathrm{T}}R & \frac{h^2}{2}H \\ \star & -I & 0 & 0 & 0 \\ \star & \star & -hZ_2 & 0 & 0 \\ \star & \star & \star & -\frac{h^2}{2}R & 0 \\ \star & \star & \star & \star & -\frac{h^2}{2}R \end{bmatrix}<0$$
$$\forall i=1,2,\dots,q \tag{7}$$

$$\Upsilon_1^{(i)}=\begin{bmatrix} X & H-\Gamma_2^{(i)\mathrm{T}} & M \\ \star & Z_1 & 0 \\ \star & \star & Z_2 \end{bmatrix}\geq 0, \quad \forall i=1,2,\dots,q \tag{8}$$

$$\Upsilon_2^{(i)}=\begin{bmatrix} X & H-\Gamma_2^{(i)\mathrm{T}} & Y \\ \star & Z_1 & 0 \\ \star & \star & Z_2 \end{bmatrix}\geq 0, \quad \forall i=1,2,\dots,q \tag{9}$$

where

$$\Xi^{(i)}=\left[\Xi_{jk}^{(i)}\right]_{5\times 5}$$
$$A_c^{(i)}=\left[A_0^{(i)} \quad 0 \quad A_1^{(i)} \quad 0 \quad B^{(i)}\right]$$
$$\Gamma_1^{(i)}=\left[L_0^{(i)}-D_fC_0^{(i)} \quad -C_f \quad L_1^{(i)}-D_fC_1^{(i)} \quad 0 \quad G^{(i)}-D_fD^{(i)}\right]$$
$$\Gamma_2^{(i)}=\big[P_4^{\mathrm{T}}A_0^{(i)}+P_5^{\mathrm{T}}B_fC_0^{(i)}+P_6 \quad P_5^{\mathrm{T}}A_f \quad P_4^{\mathrm{T}}A_1^{(i)}+P_5^{\mathrm{T}}B_fC_1^{(i)}$$
$$-P_6 \quad P_4^{\mathrm{T}}B^{(i)}+P_5^{\mathrm{T}}B_fD^{(i)}\big]$$

with

$$\Xi_{11}^{(i)}=P_1A_0^{(i)}+A_0^{(i)\mathrm{T}}P_1+P_2B_fC_0^{(i)}+C_0^{(i)\mathrm{T}}B_f^{\mathrm{T}}P_2^{\mathrm{T}}+P_4+P_4^{\mathrm{T}}$$
$$+Q_1+Q_2+hZ_1+M_1+M_1^{\mathrm{T}}+hH_1+hH_1^{\mathrm{T}}$$
$$\Xi_{12}^{(i)}=P_2A_f+A_0^{(i)\mathrm{T}}P_2+C_0^{(i)\mathrm{T}}B_f^{\mathrm{T}}P_3+P_5^{\mathrm{T}}+M_2^{\mathrm{T}}+hH_2^{\mathrm{T}}$$
$$\Xi_{13}^{(i)}=P_1A_1^{(i)}+P_2B_fC_1^{(i)}-M_1+Y_1+M_3^{\mathrm{T}}+hH_3^{\mathrm{T}}$$
$$\Xi_{14}^{(i)}=-P_4-Y_1+M_4^{\mathrm{T}}+hH_4^{\mathrm{T}}$$
$$\Xi_{15}^{(i)}=P_1B^{(i)}+P_2B_fD^{(i)}+M_5^{\mathrm{T}}+hH_5^{\mathrm{T}}$$

$$\Xi_{22}^{(i)} = P_3 A_f + A_f^{\mathrm{T}} P_3$$
$$\Xi_{23}^{(i)} = P_2^{\mathrm{T}} A_1^{(i)} + P_3 B_f C_1^{(i)} - M_2 + Y_2$$
$$\Xi_{24}^{(i)} = -P_5 - Y_2$$
$$\Xi_{25}^{(i)} = P_2^{\mathrm{T}} B^{(i)} + P_3 B_f D^{(i)}$$
$$\Xi_{33}^{(i)} = -(1-\mu) Q_2 - M_3 - M_3^{\mathrm{T}} + Y_3 + Y_3^{\mathrm{T}}$$
$$\Xi_{34}^{(i)} = -Y_3 - M_4^{\mathrm{T}} + Y_4^{\mathrm{T}}$$
$$\Xi_{35}^{(i)} = -M_5^{\mathrm{T}} + Y_5^{\mathrm{T}}$$
$$\Xi_{44}^{(i)} = -Q_1 - Y_4 - Y_4^{\mathrm{T}}$$
$$\Xi_{45}^{(i)} = -Y_5^{\mathrm{T}}$$
$$\Xi_{55}^{(i)} = -\gamma^2 I$$

Proof Construct a Lyapunov functional candidate as

$$\begin{aligned} V(x_t) &= \zeta^{\mathrm{T}}(t) P \zeta(t) + \int_{t-h}^{t} x^{\mathrm{T}}(s) Q_1 x(s)\, ds + \int_{t-d(t)}^{t} x^{\mathrm{T}}(s) Q_2 x(s)\, ds \\ &\quad + \int_{-h}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s) Z_1 x(s)\, ds\, d\theta + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) Z_2 \dot{x}(s)\, ds\, d\theta \\ &\quad + \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^{\mathrm{T}}(s) R \dot{x}(s)\, ds\, d\lambda\, d\theta \end{aligned} \tag{10}$$

where

$$\zeta(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \\ \int_{t-h}^{t} x(s)\, ds \end{bmatrix}$$

Taking the time derivative of $V(x_t)$ along the trajectory of system (6) yields

$$\begin{aligned} \dot{V}(x_t) &= 2\zeta^{\mathrm{T}}(t) P \dot{\zeta}(t) + x^{\mathrm{T}}(t)(Q_1 + Q_2 + hZ_1) x(t) \\ &\quad - x^{\mathrm{T}}(t-h) Q_1 x(t-h) - \big(1 - \dot{d}(t)\big) x^{\mathrm{T}}\big(t-d(t)\big) Q_2 x\big(t-d(t)\big) \\ &\quad + \dot{x}^{\mathrm{T}}(t)\left(hZ_2 + \frac{h^2}{2} R\right)\dot{x}(t) - \int_{t-h}^{t} x^{\mathrm{T}}(s) Z_1 x(s)\, ds \\ &\quad - \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(s) Z_2 \dot{x}(s)\, ds - \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R \dot{x}(s)\, ds\, d\theta \end{aligned} \tag{11}$$

It is easy to see that the following equalities hold:

$$2\xi^{\mathrm{T}}(t)M\left[x(t)-x\bigl(t-d(t)\bigr)-\int_{t-d(t)}^{t}\dot{x}(s)\,ds\right]=0 \tag{12}$$

$$2\xi^{\mathrm{T}}(t)Y\left[x\bigl(t-d(t)\bigr)-x(t-h)-\int_{t-h}^{t-d(t)}\dot{x}(s)\,ds\right]=0 \tag{13}$$

$$2\xi^{\mathrm{T}}(t)H\left[hx(t)-\int_{t-h}^{t-d(t)}x(s)\,ds-\int_{t-d(t)}^{t}x(s)\,ds-\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,ds\,d\theta\right]=0 \tag{14}$$

$$h\xi^{\mathrm{T}}(t)X\xi(t)-\int_{t-d(t)}^{t}\xi^{\mathrm{T}}(t)X\xi(t)\,ds-\int_{t-h}^{t-d(t)}\xi^{\mathrm{T}}(t)X\xi(t)\,ds=0 \tag{15}$$

where $\xi^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t)\ \hat{x}^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-d(t))\ x^{\mathrm{T}}(t-h)\ \omega^{\mathrm{T}}(t)]$. And,

$$\begin{aligned}&-2\xi^{\mathrm{T}}(t)H\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,ds\,d\theta\\&\quad\le\frac{1}{2}h^{2}\xi^{\mathrm{T}}(t)HR^{-1}H^{\mathrm{T}}\xi(t)+\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R_{1}\dot{x}(s)\,ds\,d\theta\end{aligned} \tag{16}$$

Adding the left sides of (12)–(15) into $\dot{V}(x_t)$ and applying (16) yields

$$\begin{aligned}&\dot{V}(x_t)-\gamma^{2}\omega^{\mathrm{T}}(t)\omega(t)+z_e^{\mathrm{T}}(t)z_e(t)\\&\quad\le\xi^{\mathrm{T}}(t)\left[\Xi+hX+\Gamma_1^{\mathrm{T}}\Gamma_1+hA_c^{\mathrm{T}}Z_2A_c+\frac{h^2}{2}A_c^{\mathrm{T}}RA_c+\frac{h^2}{2}HR^{-1}H^{\mathrm{T}}\right]\xi(t)\\&\qquad-\int_{t-\tau(t)}^{t}\xi^{\mathrm{T}}(t,s)\Upsilon_1\xi(t,s)\,ds-\int_{t-h}^{t-\tau(t)}\xi^{\mathrm{T}}(t,s)\Upsilon_2\xi(t,s)\,ds\end{aligned} \tag{17}$$

By Schur complements, (7) implies $\Xi+hX+\Gamma_1^{\mathrm{T}}\Gamma_1+hA_c^{\mathrm{T}}Z_2A_c+\frac{h^2}{2}A_c^{\mathrm{T}}RA_c+\frac{h^2}{2}HR^{-1}H^{\mathrm{T}}<0$ over the entire uncertain domain Σ. Clearly, (7)–(9) guarantee $\dot{V}(t)-\gamma^{2}\omega^{\mathrm{T}}(t)\omega(t)+z_e^{\mathrm{T}}(t)z_e(t)<0$ over the entire uncertain domain Σ. This also implies that augmented system (6) is asymptotically stable with $\omega(t)=0$.

It can also be seen that

$$\int_0^{\infty}\bigl[z_e^{\mathrm{T}}(t)z_e(t)-\gamma^{2}\omega^{\mathrm{T}}(t)\omega(t)\bigr]\,dt\le V(t)|_{t=0}-V(t)|_{t\to\infty}$$

under the zero initial condition, $V(t)|_{t=0}=0$, one can obtain

$$\int_0^{\infty}\bigl[z_e^{\mathrm{T}}(t)z_e(t)-\gamma^{2}\omega^{\mathrm{T}}(t)\omega(t)\bigr]\,dt<0$$

which implies $\|z_e(t)\|_2<\gamma\|\omega(t)\|_2$. This completes the proof. □

Remark 1 Unlike the Lyapunov functionals in the existing literature [1, 3–11, 15, 17, 18, 21, 22, 24, 26], the one introduced in this paper contains a triple integral term, that is, $\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,ds\,d\lambda\,d\theta$. In order to estimate the upper bound of the derivative of the new Lyapunov functional, (14) and (16) are introduced.

For Case 2, a similar result is concluded in the following theorem.

Theorem 2 *For given scalars $h>0$ and $\gamma>0$, the augmented system (6) is asymptotically stable with the H_∞ performance $\|z_e(t)\|_2<\gamma\|\omega(t)\|_2$ for all the delays satisfying (4) and for all the parameters that belong to the uncertain polytope (2), if there exist some matrices*

$$P=\begin{bmatrix} P_1 & P_2 & P_4\\ \star & P_3 & P_5\\ \star & \star & P_6\end{bmatrix}>0,\qquad Q_1>0,\qquad Z_1>0,\qquad Z_2>0,\qquad R>0$$

$$X=[X_{ij}]_{5\times 5}\geq 0,\qquad Y=\begin{bmatrix} Y_1\\ Y_2\\ Y_3\\ Y_4\\ Y_5\end{bmatrix},\qquad M=\begin{bmatrix} M_1\\ M_2\\ M_3\\ M_4\\ M_5\end{bmatrix},\qquad H=\begin{bmatrix} H_1\\ H_2\\ H_3\\ H_4\\ H_5\end{bmatrix}$$

with appropriate dimensions such that

$$\tilde{\Phi}^{(i)}=\begin{bmatrix} \tilde{\Xi}^{(i)}+hX & \Gamma_1^{(i)\mathrm{T}} & hA_c^{(i)\mathrm{T}}Z_2 & \frac{h^2}{2}A_c^{(i)\mathrm{T}}R & \frac{h^2}{2}H\\ \star & -I & 0 & 0 & 0\\ \star & \star & -hZ_2 & 0 & 0\\ \star & \star & \star & -\frac{h^2}{2}R & 0\\ \star & \star & \star & \star & -\frac{h^2}{2}R\end{bmatrix}<0$$

$$\forall i=1,2,\ldots,q \tag{18}$$

$$\Upsilon_1^{(i)}=\begin{bmatrix} X & H-\Gamma_2^{(i)\mathrm{T}} & M\\ \star & Z_1 & 0\\ \star & \star & Z_2\end{bmatrix}\geq 0,\quad \forall i=1,2,\ldots,q \tag{19}$$

$$\Upsilon_2^{(i)}=\begin{bmatrix} X & H-\Gamma_2^{(i)\mathrm{T}} & Y\\ \star & Z_1 & 0\\ \star & \star & Z_2\end{bmatrix}\geq 0,\quad \forall i=1,2,\ldots,q \tag{20}$$

where

$$\tilde{\Xi}^{(i)}=[\tilde{\Xi}_{jk}]_{5\times 5}$$

$$\tilde{\Xi}_{11}^{(i)}=\Xi_{11}^{(i)}-Q_2$$

$$\tilde{\Xi}_{33}^{(i)}=\Xi_{33}^{(i)}+(1-\mu)Q_2$$

where $\tilde{\Xi}_{jk}$, $1 \le j \le k \le 5$, $(j,k) \ne (1,1)$ or $(3,3)$, $\Gamma_1^{(i)}$, $\Gamma_2^{(i)}$, $A_c^{(i)}$ *are the same as those defined in Theorem* 1.

Proof Choose a Lyapunov functional candidate as

$$V(x_t) = \zeta^{\mathrm{T}}(t)P\zeta(t) + \int_{t-h}^{t} x^{\mathrm{T}}(s)Q_1x(s)\,ds + \int_{-h}^{0}\int_{t+\theta}^{t} x^{\mathrm{T}}(s)Z_1x(s)\,ds\,d\theta + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)\,ds\,d\theta + \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\,ds\,d\lambda\,d\theta \quad (21)$$

Following a line similar to that of Theorem 1, the proof can be completed. □

Remark 2 If we let $P_4 = 0$, $P_5 = 0$, $M_4 = 0$, $H = 0$, $Y = 0$, $P_6 = \epsilon_1 I$, $Z_1 = \epsilon_2 I$, $Q_1 = \epsilon_3 I$, and $R = \epsilon_4 I$ with $\epsilon_i > 0$, $i = 1,2,3,4$ being some sufficient small scalars, Theorem 2 yields an equivalent form of Corollary 5 in [26]. The proof can be completed following a similar procedure as in [23] and is omitted here. Therefore, these variables can provide extra freedom in Theorem 2. It may be expected that Theorem 2 is less conservative than Corollary 5 in [26].

3.2 H_∞ Filter Design

In the following part, the method of the design of the filer parameters $\{A_f, B_f, C_f, D_f\}$ is developed. These parameters can be obtained by solving a set of linear matrix inequalities (LMIs). For Case 1, the following theorem is obtained.

Theorem 3 *For given scalars* $h > 0$, $\gamma > 0$, μ *and* ε, *an* H_∞ *filter of the form* (5) *for system* (1) *exists if there exist matrices* $P_1 > 0$, $P_6 > 0$, $T > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z_1 > 0$, $Z_2 > 0$, $R > 0$, P_4, N_j, $j = 1,2,3,4$, $\hat{X} = [\hat{X}_{ij}]_{5\times 5} \ge 0$,

$$Y = \begin{bmatrix} Y_1 \\ \hat{Y}_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix}, \qquad M = \begin{bmatrix} M_1 \\ \hat{M}_2 \\ M_3 \\ M_4 \\ M_5 \end{bmatrix}, \qquad H = \begin{bmatrix} H_1 \\ \hat{H}_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix}$$

with appropriate dimensions such that

$$\begin{bmatrix} P_1 - T & 0 & P_4 - \varepsilon T \\ \star & T & \varepsilon T \\ \star & \star & P_6 \end{bmatrix} > 0 \quad (22)$$

$$\Theta^{(i)} = \begin{bmatrix} \Omega^{(i)} + h\hat{X} & \Lambda_1^{(i)\mathrm{T}} & hA_c^{(i)\mathrm{T}}Z_2 & \frac{h^2}{2}A_c^{(i)\mathrm{T}}R & \frac{h^2}{2}H \\ \star & -I & 0 & 0 & 0 \\ \star & \star & -hZ_2 & 0 & 0 \\ \star & \star & \star & -\frac{h^2}{2}R & 0 \\ \star & \star & \star & \star & -\frac{h^2}{2}R \end{bmatrix} < 0$$

$$\forall i = 1,2,\ldots,q \quad (23)$$

$$\Pi_1^{(i)} = \begin{bmatrix} \hat{X} & H - \Lambda_2^{(i)\mathrm{T}} & M \\ \star & Z_1 & 0 \\ \star & \star & Z_2 \end{bmatrix} \geq 0, \quad \forall i = 1, 2, \ldots, q \tag{24}$$

$$\Pi_2^{(i)} = \begin{bmatrix} \hat{X} & H - \Lambda_2^{(i)\mathrm{T}} & Y \\ \star & Z_1 & 0 \\ \star & \star & Z_2 \end{bmatrix} \geq 0, \quad \forall i = 1, 2, \ldots, q \tag{25}$$

where

$$\Omega^{(i)} = \big[\Omega_{jk}^{(i)}\big]_{5\times 5}$$

$$A_c^{(i)} = \big[A_0^{(i)} \quad 0 \quad A_1^{(i)} \quad 0 \quad B^{(i)}\big]$$

$$\Lambda_1^{(i)} = \big[L_0^{(i)} - N_4 C_0^{(i)} \quad -N_3 \quad L_1^{(i)} - N_4 C_1^{(i)} \quad 0 \quad G^{(i)} - N_4 D^{(i)}\big]$$

$$\begin{aligned}\Lambda_2^{(i)} = \big[&P_4^{\mathrm{T}} A_0^{(i)} + \varepsilon N_2 C_0^{(i)} + P_6 \quad \varepsilon N_1 \quad P_4^{\mathrm{T}} A_1^{(i)} + \varepsilon N_2 C_1^{(i)} \\ &-P_6 \ P_4^{\mathrm{T}} B^{(i)} + \varepsilon N_2 D^{(i)}\big]\end{aligned}$$

with

$$\begin{aligned}
\Omega_{11}^{(i)} &= P_1 A_0^{(i)} + A_0^{(i)\mathrm{T}} P_1 + N_2 C_0^{(i)} + C_0^{(i)\mathrm{T}} N_2^{\mathrm{T}} + P_4 + P_4^{\mathrm{T}} + Q_1 + Q_2 + hZ_1 \\
&\quad + M_1 + M_1^{\mathrm{T}} + hH_1 + hH_1^{\mathrm{T}} \\
\Omega_{12}^{(i)} &= N_1 + A_0^{(i)\mathrm{T}} T + C_0^{(i)\mathrm{T}} N_2^{\mathrm{T}} + \varepsilon T + \hat{M}_2^{\mathrm{T}} + h\hat{H}_2^{\mathrm{T}} \\
\Omega_{13}^{(i)} &= P_1 A_1^{(i)} + N_2 C_1^{(i)} - M_1 + Y_1 + M_3^{\mathrm{T}} + hH_3^{\mathrm{T}} \\
\Omega_{14}^{(i)} &= -P_4 - Y_1 + M_4^{\mathrm{T}} + hH_4^{\mathrm{T}} \\
\Omega_{15}^{(i)} &= P_1 B^{(i)} + N_2 D^{(i)} + M_5^{\mathrm{T}} + hH_5^{\mathrm{T}} \\
\Omega_{22}^{(i)} &= N_1 + N_1^{\mathrm{T}} \\
\Omega_{23}^{(i)} &= T A_1^{(i)} + N_2 C_1^{(i)} - \hat{M}_2 + \hat{Y}_2 \\
\Omega_{24}^{(i)} &= -\varepsilon T - \hat{Y}_2 \\
\Omega_{25}^{(i)} &= T B^{(i)} + N_2 D^{(i)} \\
\Omega_{33}^{(i)} &= -(1-\mu) Q_2 - M_3 - M_3^{\mathrm{T}} + Y_3 + Y_3^{\mathrm{T}} \\
\Omega_{34}^{(i)} &= -Y_3 - M_4^{\mathrm{T}} + Y_4^{\mathrm{T}} \\
\Omega_{35}^{(i)} &= -M_5^{\mathrm{T}} + Y_5^{\mathrm{T}} \\
\Omega_{44}^{(i)} &= -Q_1 - Y_4 - Y_4^{\mathrm{T}} \\
\Omega_{45}^{(i)} &= -Y_5^{\mathrm{T}} \\
\Omega_{55}^{(i)} &= -\gamma^2 I
\end{aligned}$$

and the filter parameters are given by $A_f = N_1T^{-1}$, $B_f = N_2$, $C_f = N_3T^{-1}$, $D_f = N_4$.

Proof Since (22) implies $T > 0$, there always exists a nonsingular matrix P_2 and a matrix $P_3 > 0$ such that $T = P_2P_3^{-1}P_2^{\mathrm{T}}$. Define $J = P_2P_3^{-1}$ and $P_5 = \varepsilon P_2^{\mathrm{T}}$, pre- and post-multiply both sides of (7) with $\mathrm{diag}\{I, J, I, I, I, I, I, I, I\}$ and its transpose, pre- and post-multiply both sides of (8)–(9) with $\mathrm{diag}\{I, J, I, I, I, I, I\}$ and its transpose, and introduce new variables $N_1 = P_2A_fP_3^{-1}P_2^{\mathrm{T}}$, $N_2 = P_2B_f$, $N_3 = C_fP_3^{-1}P_2^{\mathrm{T}}$, $N_4 = D_f$, $\hat{M}_2 = P_2P_3^{-1}M_2$, $\hat{Y}_2 = P_2P_3^{-1}Y_2$, $\hat{H}_2 = P_2P_3^{-1}H_2$,

$$\hat{X} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & J & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} X \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & J^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

then (23)–(25) will be obtained.

Pre- and post-multiplying both sides of

$$\begin{bmatrix} P_1 & P_2 & P_4 \\ \star & P_3 & P_5 \\ \star & \star & P_6 \end{bmatrix} > 0 \quad \text{with} \begin{bmatrix} I & -J & 0 \\ 0 & J & 0 \\ 0 & 0 & I \end{bmatrix}$$

and its transpose, one can obtain (22) directly.

It is easy to see that the filter of the form (5) with $A_f = P_2^{-1}N_1T^{-1}P_2$, $B_f = P_2^{-1}N_2$, $C_f = N_3T^{-1}P_2$, $D_f = N_4$ can guarantee that the augmented system (6) is asymptotically stable with the H_∞ performance $\|z_e(t)\|_2 < \gamma\|\omega(t)\|_2$.

The filter transfer function from $y(t)$ to $\hat{z}(t)$ can be obtained by

$$\begin{aligned} T_{\hat{z}y} &= C_f(sI - A_f)^{-1}B_f + D_f \\ &= N_3T^{-1}P_2\big(sI - P_2^{-1}N_1T^{-1}P_2\big)^{-1}P_2^{-1}N_2 + N_4 \\ &= N_3T^{-1}\big(sI - N_1T^{-1}\big)^{-1}N_2 + N_4 \end{aligned}$$

So, the filter parameters are given by $A_f = N_1T^{-1}$, $B_f = N_2$, $C_f = N_3T^{-1}$, $D_f = N_4$. This completes the proof. □

For Case 2, a similar result is obtained in the following theorem.

Theorem 4 *For given scalars* $h > 0$, $\gamma > 0$, *and* ε, *an* H_∞ *filter of the form* (5) *for system* (1) *exists if there exist matrices* $P_1 > 0$, $P_6 > 0$, $T > 0$, $Q_1 > 0$, $Z_1 > 0$,

$Z_2 > 0$, $R > 0$, P_4, N_j, $j = 1, 2, 3, 4$, $\hat{X} = [\hat{X}_{ij}]_{5\times 5} \geq 0$,

$$Y = \begin{bmatrix} Y_1 \\ \hat{Y}_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix}, \qquad M = \begin{bmatrix} M_1 \\ \hat{M}_2 \\ M_3 \\ M_4 \\ M_5 \end{bmatrix}, \qquad H = \begin{bmatrix} H_1 \\ \hat{H}_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix}$$

such that

$$\begin{bmatrix} P_1 - T & 0 & P_4 - \varepsilon T \\ \star & T & \varepsilon T \\ \star & \star & P_6 \end{bmatrix} > 0 \tag{26}$$

$$\tilde{\Theta}^{(i)} = \begin{bmatrix} \tilde{\Omega}^{(i)} + h\hat{X} & \Lambda_1^{(i)\mathrm{T}} & hA_c^{(i)\mathrm{T}} Z_2 & \frac{h^2}{2} A_c^{(i)\mathrm{T}} R & \frac{h^2}{2} H \\ \star & -I & 0 & 0 & 0 \\ \star & \star & -hZ_2 & 0 & 0 \\ \star & \star & \star & -\frac{h^2}{2} R & 0 \\ \star & \star & \star & \star & -\frac{h^2}{2} R \end{bmatrix} < 0$$

$$\forall i = 1, 2, \ldots, q \tag{27}$$

$$\tilde{\Pi}_1^{(i)} = \begin{bmatrix} \hat{X} & H - \Lambda_2^{(i)\mathrm{T}} & M \\ \star & Z_1 & 0 \\ \star & \star & Z_2 \end{bmatrix} \geq 0, \quad \forall i = 1, 2, \ldots, q \tag{28}$$

$$\tilde{\Pi}_2^{(i)} = \begin{bmatrix} \hat{X} & H - \Lambda_2^{(i)\mathrm{T}} & Y \\ \star & Z_1 & 0 \\ \star & \star & Z_2 \end{bmatrix} \geq 0, \quad \forall i = 1, 2, \ldots, q \tag{29}$$

where

$$\tilde{\Omega}^{(i)} = [\tilde{\Omega}_{jk}^{(i)}]_{5\times 5}$$

$$\tilde{\Omega}_{11}^{(i)} = \Omega_{11}^{(i)} - Q_2$$

$$\tilde{\Omega}_{33}^{(i)} = \Omega_{33}^{(i)} + (1 - \mu) Q_2$$

and $\tilde{\Omega}_{jk}$, $1 \leq j \leq k \leq 5$, $(j,k) \neq (1,1)$ or $(3,3)$, $\Lambda_1^{(i)}$, $\Lambda_2^{(i)}$, $A_c^{(i)}$ *are the same as those defined in Theorem 3. The filter parameters are given by* $A_f = N_1 T^{-1}$, $B_f = N_2$, $C_f = N_3 T^{-1}$, $D_f = N_4$.

Remark 3 A tuning parameter ε is involved in Theorems 3 and 4. Similar to [27], a tuning procedure for ε is presented. Choose a cost function to be $f(\varepsilon) = t_{\min}$. The parameter $t_{\min}$ is obtained by solving the feasibility problem using an LMI toolbox. Then, apply a numerical optimization algorithm, such as **fminsearch** in the optimization toolbox of MATLAB, to $f(\varepsilon) = t_{\min}$ and a locally convergent feasible solution will be obtained. If the resulting minimum value of the cost function is negative, then an appropriate tuning parameter ε is found. Some numerical examples have shown that $\varepsilon = 0$ may often yield a satisfactory result.

4 Numerical Examples

In this section, two numerical examples are given to show the effectiveness and advantage of the proposed method.

Example 1 Consider system (1) with

$$A_0 = \begin{bmatrix} 0 & 2 \\ -1 & -2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.1 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_0 = [\,1 \quad 0\,]$$
$$C_1 = [\,0 \quad 1\,], \quad L_0 = [\,1 \quad 2\,], \quad L_1 = [\,0 \quad 1\,], \quad D = 1, \quad G = 0$$

The objective is to calculate the minimum H_∞ performance $\gamma_{\min}$ when the delay is bounded by 1, that is, $h = 1$. Results compared with those in [26] are listed in Table 1 for different values of μ or unknown μ.

On the other hand, the upper bound on the delay can be obtained if the γ is prescribed. Setting $\gamma = 0.5$, the upper bounds on the delay for different values of μ or unknown μ are listed in Table 2.

From these two tables, it can be seen that our method can lead to a much less conservative result. For $\gamma = 0.6123$, $h = 1$, $\mu = 0.5$, the filter gains can be obtained as

$$A_f = \begin{bmatrix} -1.4604 & 1.9599 \\ -1.8094 & -0.6524 \end{bmatrix}, \qquad B_f = \begin{bmatrix} -0.3680 \\ -1.5530 \end{bmatrix}$$
$$C_f = [\,-0.1700 \quad -0.9414\,], \qquad D_f = 0.2723$$

For the purpose of simulation, the external disturbance signal is assumed to be $\omega(t) = \frac{1}{t^3+1}$. The estimation error of $z(t)$ is shown in Fig. 1. It can be seen that

Table 1 Minimum H_∞ performance $\gamma_{\min}$ for different μ with $h = 1$

μ	0.1	0.3	0.5	0.7	0.9	Unknown
Zhang and Han [26]	0.5278	0.5742	0.6362	0.7295	0.8893	1.0001
Our results	0.5003	0.5477	0.6123	0.6778	0.6872	0.6872

Table 2 Maximum upper bound h for different μ with $\gamma = 0.5$

μ	0.1	0.3	0.5	0.7	0.9	Unknown
Zhang and Han [26]	0.8369	0.6655	0.5584	0.5019	0.4826	0.4819
Our results	0.9987	0.7997	0.6972	0.6772	0.6772	0.6672

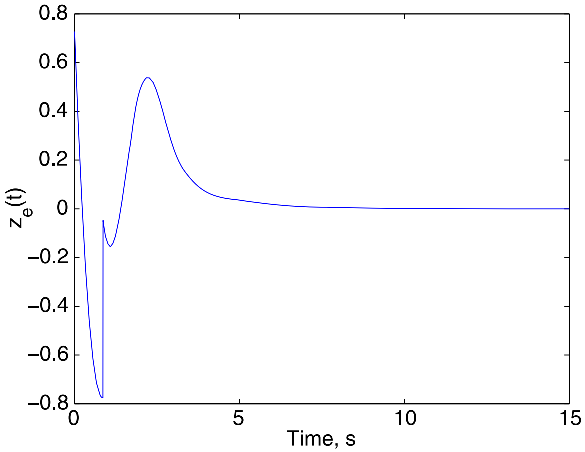

Fig. 1 Estimation error of $z(t)$ for $\gamma = 0.6123$, $h = 1$, $\mu = 0.5$

Table 3 Maximum upper bound h for different μ with $\gamma = 5$

μ	0.2	0.4	0.6	0.8	Unknown
Gao and Wang [6]	0.4213	0.3054	0.1944	0.0923	–
Zhang and Han [26]	0.6174	0.5765	0.5344	0.4894	0.4420
Our results	0.6521	0.6393	0.6366	0.6366	0.6366

the error system is asymptotically stable, which illustrates the effectiveness of the proposed method in this paper.

Example 2 Consider system (1) with

$$A_0 = \begin{bmatrix} -2 & 0 \\ 0 & -0.7+\rho(t) \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1 & -1+\sigma(t) \\ -1 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.5 \\ 2 \end{bmatrix}$$

$$C_0 = [\,0 \quad 1\,], \qquad C_1 = [\,1 \quad 2\,], \qquad L_0 = [\,2 \quad 1\,]$$

$$L_1 = [\,0 \quad 0\,], \qquad D = 1, \qquad G = 0$$

The uncertain parameters satisfy $|\rho(t)| \le 0.2$ and $|\sigma(t)| \le 0.5$.

Assume that the time-varying delay satisfies Case 1. In order to compare with the results in [5, 26], D_f is assumed to be 0. Applying Theorem 1, the maximum delay bound, h, can be obtained for different values of μ, under a given H_∞ performance $\gamma = 5$. The results compared with those obtained in [5, 26] are listed in Table 3. On the other hand, the minimum achievable value of γ can be calculated for a given $h = 0.44$. Table 4 lists the results compared with those obtained in [5, 26]. It can be seen that our method can lead to much bigger maximum delay bounds when the performance level γ is given. On the other hand, our method can lead to a much smaller performance level γ when the delay bound is given.

Table 4 Minimum H_∞ performance $\gamma_{\min}$ for different μ with $h = 0.44$

μ	0.2	0.4	0.6	0.8	Unknown
Gao and Wang [6]	5.9416	–	–	–	–
Zhang and Han [26]	1.6631	1.7819	2.0104	2.4557	4.2334
Our results	1.4419	1.4419	1.4419	1.4419	1.4419

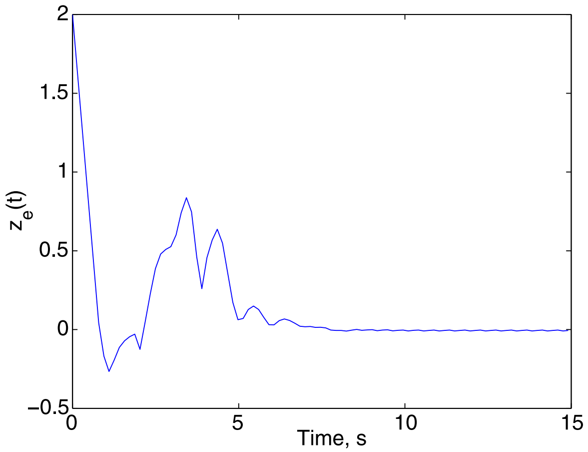

Fig. 2 Estimation error of $z(t)$ for $\gamma = 5$, $h = 0.6366$, $\mu = 0.8$

For $\gamma = 5$, $h = 0.6366$, $\mu = 0.8$, the filter gains can be obtained as

$$A_f = 10^4 * \begin{bmatrix} -0.8622 & -2.5342 \\ -0.4341 & -1.2766 \end{bmatrix}, \qquad B_f = \begin{bmatrix} 2.0740 \\ -0.9429 \end{bmatrix}$$
$$C_f = 10^4 * [-0.4326 \quad -1.2719]$$

For the purpose of simulation, the external disturbance signal is assumed to be

$$\omega(t) = \begin{cases} 1, & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases}$$

The estimation error of $z(t)$ is shown as in Fig. 2. It can be seen that the error system is asymptotically stable, which demonstrates the effectiveness of the proposed method in this paper.

5 Conclusions

The problem of H_∞ filtering for systems with time-varying delay and polytopic uncertainties has been investigated in this paper. Two kinds of time delays have been considered and new sufficient conditions for the existences of the H_∞ filters have been derived in terms of LMIs on the basis of introducing a new Lyapunov functional.

A filter design method has also been proposed. Numerical examples have shown that the method proposed in this paper is effective and can lead to less conservative results.

This paper has only considered the asymptotical stability of the error state system. The exponential stability problem will be a future research topic. It should be noted that our method can be extended to more general systems such as nonlinear and/or stochastic systems with distributed delays, and the results reported in [14, 19, 20] may be beneficial to further investigations.

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