

# NETWORKED PREDICTIVE CONTROL FOR HAMMERSTEIN SYSTEMS

Jian Sun, Guo-Ping Liu, Jie Chen, and David Rees

## ABSTRACT

This paper is concerned with the problems of design and stability analysis of networked predictive control for Hammerstein systems. The Hammerstein nonlinearity is removed (or partially removed) by inverting it. By predicting the future control sequence, the random network-induced delay and data dropout are compensated actively. The stability of the closed-loop system is analyzed by applying the switched Lyapunov function approach. Simulation results are presented to illustrate the validity of the proposed method.

**Key Words:** Networked control systems, Hammerstein systems, networked predictive control, switched Lyapunov function, network-induced delay.

## I. INTRODUCTION

Feedback control systems whose control loops are connected with real-time networks to exchange information and control signals are labeled as networked control systems (NCSs). NCSs have received much attention due to many practical advantages such as reduced wiring, low power requirements, and flexibility of operations [1, 2]. On the other hand, network insertion also brings about negative effects such as the network-induced delay and data dropout which present challenges to traditional control theories. To attempt to solve these problems, various control methods have

been proposed [3–11]. For example, the modeling and analysis of MIMO NCSs with multiple time-varying delays were investigated in [3]. In [4], a model-based control method was proposed whose key idea is using the knowledge of plant dynamics to reduce the traffic burden of the network. In [5], the stability of nonlinear NCSs was studied and an important concept MATI (maximum allowable transfer interval) was proposed. NCSs were modeled as time-delay systems in [6, 7] taking the networked delay and data dropout into account. Based on the Lyapunov stability theory, the stability and stabilization of NCSs were also investigated. As for other methods, please refer to the recent survey paper [12] and references therein.

Recently, a networked predictive method has been proposed in [13, 14]. Numerical simulations and practical experiments have illustrated that this method can adequately compensate for the network-induced delay and data dropout. In this paper, we extend this method to networked Hammerstein systems. The Hammerstein model, where a linear time-invariant system is preceded by a static input nonlinearity, is a typical block-oriented nonlinear model. In many cases, Hammerstein systems present a good tradeoff between the complexity of nonlinear systems and the interpretability of linear dynamical systems [15]. A complicated nonlinear plant can be broken down where parts of the plant include a Hammerstein system. Hammerstein systems have

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been used to model PH neutralization processes [16], electrical drives for online testing [17], solid oxide fuel cells [18], vestibulo-ocular reflexes (VOR) [19], and acoustic echo cancelation [20]. So it is important, in both theory and application, to study the control method for networked Hammerstein systems. In [16], the model predictive control for Hammerstein systems was considered. The key idea in [16] is that the input nonlinearity is removed from the control problem by inverting it. If the inverse of the nonlinear function is obtained accurately, the Hammerstein system is stable if, and only if, its linear part is stable. So a linear model predictive control method can be used to design the controller for Hammerstein systems. This idea is adopted in this paper to design a controller for networked Hammerstein systems.

It should be noted that the predictive control of networked systems based on the Hammerstein model has been considered in [21, 22]. However, [21] has only investigated the stability of the system when the network-induced delay is constant, which is not always the case in practical applications. In [22], the control increment, instead of the control signal, is obtained by the predictive control approach, which complicates the application of the method since the past control increment should be known at the controller node. Furthermore, the method cannot removed the nonlinearity essentially since  $f(\Delta u(k)) \neq f(u(k)) - f(u(k-1))$  even if the inverse of the nonlinear function is obtained accurately. And, the stability analysis is on the basis of the single common quadratic Lyapunov function approach which is well-known to give a conservative condition.

In this paper, a model predictive control based strategy is proposed to compensate for the network-induced delay and data dropout. It should be noted that the control signal but not the control increment is obtained in this approach by the delayed system states which is always available for the controller node. Therefore, this approach is easy to implement. In order to deal with the input nonlinearity, we first design a model predictive controller for the linear part of the Hammerstein system, and an intermediate control signal  $v(k)$  is obtained. Secondly, the control signal  $u(k)$  is calculated by inverting the static input nonlinearity, that is,  $u(k) = f^{-1}(v(k))$ . If the inverse of the nonlinearity is obtained accurately, the nonlinearity will be essentially removed from the control problem. The stability of the closed-loop systems is analyzed using the switched Lyapunov function approach when the inverse of the nonlinearity is accurate or inaccurate. A numerical example is given to show the feasibility and efficiency of the proposed method.

## II. NETWORKED PREDICTIVE CONTROL SCHEME

Consider the following single-input-single-output Hammerstein system:

$$\begin{aligned} x(k+1) &= Ax(k) + bv(k) \\ y(k) &= cx(k) \\ v(k) &= f(u(k)) \end{aligned} \quad (1)$$

where  $x \in \mathcal{R}^n$ ,  $u, v, y \in \mathcal{R}$ , and  $f(\cdot): \mathcal{R} \rightarrow \mathcal{R}$  is a memoryless static nonlinear function.  $A$ ,  $b$ , and  $c$  are system matrices with appropriate dimensions.

The above Hammerstein system is controlled over the network. The network-induced delay and data dropout are two main issues in networked control systems. In a typical model predictive control application, a sequence of forward control signals is obtained at each sampling instant and only the current control signal is applied to the plant and the remaining predicted control signals are discarded. Note that the model predictive controller can determine future control signals and the network can transmit a set of data at the same time, therefore, the model predictive control method can be modified to compensate for the network-induced delay and data dropout.

In this paper, it is assumed that all the packets in the network are transmitted with time-stamps. On the controller node, the controller receives sensor signals subject to network transmission delay and data dropout. Using these signals, the controller generates a sequence of control prediction  $[u(k|k - \tau_{sc}^k), \dots, u(k - \tau_{sc}^k + N_u - 1|k - \tau_{sc}^k)]$  by the model predictive control method. The sequence of control prediction is packed into one packet and sent to the actuator node. On the actuator node, a buffer is set to store the sequence of control prediction. The actuator receives this packet and compares it with the one already stored in the buffer according to the time-stamp. If the packet just arrived is 'newer' than that stored in the buffer, it replaces the one in the buffer, otherwise it will be discarded. As a result of this comparison, the sequence of the control prediction stored in the buffer is always the latest one. The network-induced delay may be random which means that the data packet that was sent earlier may arrive later, and vice versa. We may refer to this phenomenon as 'packet disorder'. After the above comparison process, packet disorder can be dealt with. According to the time-delay subjected by the packet in its transmission from the controller to the actuator, the actuator selects the appropriate control signal. For example, if the time-delay from the controller to the actuator is  $\tau_{ca}$ , so the

appropriate control signal should be  $u(k + \tau_{ca}|k - \tau_{sc}^k)$ . Clearly, using the above scheme, the network-induced delay is compensated actively. In the above scheme, data dropout is not seen as an infinite delay but ignored. If the sum of the maximum round-trip delay (defined as  $\bar{\tau} = \bar{\tau}_{sc} + \bar{\tau}_{ca}$ ) and the maximum number of the continuous data dropout (denoted as  $\iota$ ) satisfies  $\bar{\tau} + \iota < N_u$ , then there will always be an appropriate control signal available at the actuator node.

In the following part, the design procedure for the control prediction for Hammerstein system is presented. The first step is to design a model predictive controller for the linear part of system (1). There usually exists a time-delay,  $\tau_{sc}^k$ , at  $k$  instant in the backward channel, so the following objective function is adopted:

$$J(k) = \|\hat{Y}(k|k - \tau_{sc}^k) - Y_r(k)\|_Q^2 + \|V(k|k - \tau_{sc}^k)\|_R^2 \quad (2)$$

where  $\hat{Y}(k|k - \tau_{sc}^k) = [\hat{y}(k - \tau_{sc}^k + 1|k - \tau_{sc}^k) \cdots \hat{y}(k - \tau_{sc}^k + N_p|k - \tau_{sc}^k)]^T$  is the predictive output,  $V(k|k - \tau_{sc}^k) = [v(k - \tau_{sc}^k|k - \tau_{sc}^k) \cdots v(k - \tau_{sc}^k + N_u - 1|k - \tau_{sc}^k)]^T$  is the intermediate control input,  $Y_r(k) = [y_r(k - \tau_{sc}^k + 1) \cdots y_r(k - \tau_{sc}^k + N_p)]^T$  is the reference signal,  $Q$  and  $R$  are constant weighting matrices,  $N_p$  and  $N_u$  are the prediction horizon and the control horizon, respectively.

The prediction for the linear part of system (1) is given by:

$$\begin{aligned} & \hat{y}(k - \tau_{sc}^k + j|k - \tau_{sc}^k) \\ &= cA^j x(k - \tau_{sc}^k) + \sum_{i=1}^j cA^{i-1}bv(k + j - i|k - \tau_{sc}^k) \end{aligned} \quad (3)$$

Therefore,

$$\hat{Y}(k|k - \tau_{sc}^k) = Fx(k - \tau_{sc}^k) + HV(k|k - \tau_{sc}^k) \quad (4)$$

where  $H$  is a block lower triangular matrix with its nonnull elements defined by  $H_{ij} = cA^{i-j}b$ , ( $j < N_u$ ), and  $H_{iN_u} = \sum_{l=0}^{i-N_u} cA^l b$ , and matrix  $F$  is defined as

$$F = \begin{bmatrix} cA \\ cA^2 \\ \vdots \\ cA^{N_p} \end{bmatrix}.$$

The optimal control sequence  $V(k|k - \tau_{sc}^k)$  is calculated by minimizing the objective function (2) and is obtained as:

$$\begin{aligned} & V(k|k - \tau_{sc}^k) \\ &= (H^T QH + R)^{-1} H^T (Y_r(k) - Fx(k - \tau_{sc}^k)) \end{aligned} \quad (5)$$

Define the intermediate control signal from  $k$  to  $k - \tau_{sc}^k + N_u - 1$  as  $V^*(k|k - \tau_{sc}^k) = [v(k|k - \tau_{sc}^k) \cdots v(k - \tau_{sc}^k + N_u - 1|k - \tau_{sc}^k)]^T$ , and thus

$$V^*(k|k - \tau_{sc}^k) = M_{\tau_{sc}^k} V(k|k - \tau_{sc}^k) \quad (6)$$

where  $M_{\tau_{sc}^k} = [0_{(N_u - \tau_{sc}^k) \times \tau_{sc}^k} \ I_{(N_u - \tau_{sc}^k) \times (N_u - \tau_{sc}^k)}]$ .

**Remark 1.** If the state vector  $x(k)$  is not all measurable, an observer should be included, which calculates the estimation by means of

$$\hat{x}(k|k) = \hat{x}(k|k - 1) + L[y_m(k) - \hat{y}(k|k - 1)]$$

where  $y_m(k)$  is the measured output.

From (5) and (6), the intermediate control sequence from  $k$  to  $k - \tau_{sc}^k + N_u - 1$  is obtained. Assuming the nonlinear function  $f(\cdot)$  is invertible and denoting its inverse  $f^{-1}(\cdot) = g(\cdot)$ , the control signal sequence is obtained as

$$\begin{aligned} & u(k + i|k - \tau_{sc}^k) = g(v(k + i|k - \tau_{sc}^k)), \\ & i = 0, \dots, N_u - \tau_{sc}^k - 1 \end{aligned} \quad (7)$$

Denote  $U^*(k|k - \tau_{sc}^k) = [u(k|k - \tau_{sc}^k) \cdots u(k - \tau_{sc}^k + N_u - 1|k - \tau_{sc}^k)]^T$ .

If  $g(\cdot)$  can be obtained accurately, the input nonlinearity can be essentially removed from the control problem. However, in practice, it is nearly impossible to calculate  $g(\cdot)$  accurately, that is,  $f(g(\cdot)) \neq 1(\cdot)$ . For this case, similar to [23], denote the practical inverse of  $f(\cdot)$  as  $\hat{g}(\cdot)$  and assume that

$$\rho_1 \theta \leq f(\hat{g}(\theta)) \leq \rho_2 \theta \quad \forall \theta \in \mathcal{R} \quad (8)$$

for some  $\rho_1 \leq \rho_2 < \infty$ . The method of how to determine  $\rho_1$  and  $\rho_2$  can be seen in [23].

The networked predictive control method for the Hammerstein systems proposed in this paper can be summarized by the following algorithm:

1. According to the sensor signal received, calculate the intermediate control sequence  $V^*(k|k - \tau_{sc}^k)$  by (5)–(6). Obtain the control sequence  $U^*(k|k - \tau_{sc}^k)$  by the inverting process (7);
2. Pack the control sequence  $U^*(k|k - \tau_{sc}^k)$  into one packet and send it to the actuator side;
3. After the comparison process, the actuator selects an appropriate control signal from the control sequence according to the delay from the controller to the actuator and apply it to the plant.

### III. STABILITY ANALYSIS

In this section, the stability of the closed-loop system is investigated. In NCSs, the network-induced delay may be random which presents a challenge to the stability analysis. Moreover, when the inverse of the input nonlinear function cannot be obtained accurately, the nonlinearity may not be removed totally. It also brings some difficulties in the stability analysis. In this section, the switched Lyapunov function approach is used to derive stability criteria for the system when the inverse of the input nonlinear function is obtained accurately or inaccurately.

Due to the comparison process, the control sequence stored in the buffer is the latest one. Assume the forward delay and backward delay that the control sequence is subject to at the instant  $k$  are  $\tau_{ca}^k$  and  $\tau_{sc}^{k-\tau_{ca}^k}$ , respectively. Denote the round trip delay as  $\tau_k = \tau_{ca}^k + \tau_{sc}^{k-\tau_{ca}^k}$ , and the control signal selected by the actuator at the instant  $k$  is:

$$\begin{aligned} u(k) &= u(k|k - \tau_k) \\ &= \mathcal{S}_{\tau_k} U^*(k - \tau_{ca}^k | k - \tau_k) \end{aligned} \quad (9)$$

where  $\mathcal{S}_{\tau_k}$  is the selection matrix with all entries being zero except the  $(\tau_{ca}^k + 1)$ th being one.

For stability analysis, it is assumed that  $Y_r(k) = 0$  without loss of generality. If  $g(\cdot)$  is obtained accurately, then

$$\begin{aligned} u(k) &= \mathcal{S}_{\tau_k} U^*(k - \tau_{ca}^k | k - \tau_k) \\ &= g(-K_{\tau_k} x(k - \tau_k)) \end{aligned} \quad (10)$$

where  $K_{\tau_k} = \mathcal{S}_{\tau_k} M_{\tau_{sc}^{k-\tau_{ca}^k}} (H^T Q H + R)^{-1} H^T F$ . Then, the closed-loop system can be obtained as:

$$\begin{aligned} x(k+1) &= Ax(k) + bf(u(k)) \\ &= Ax(k) + bf(g(-K_{\tau_k} x(k - \tau_k))) \\ &= Ax(k) - bK_{\tau_k} x(k - \tau_k) \end{aligned} \quad (11)$$

Define  $\xi(k) = [x^T(k) x^T(k-1) \cdots x^T(k-\bar{\tau})]^T$  and rewrite (11) as:

$$\xi(k+1) = \Lambda_{\tau_k} \xi(k) \quad (12)$$

where

$$\Lambda_{\tau_k} = \begin{bmatrix} A & 0 & \cdots & -bK_{\tau_k} & 0 & \cdots & 0 & 0 \\ & & & & & & & 0 \\ & & & & & & & \vdots \\ & & & & & & & 0 \end{bmatrix}$$

The position and value of the term  $-bK_{\tau_k}$  depends on the delays in the backward and forward channel. So, system (12) is a typical switched system. The number of the subsystems is  $\bar{\tau} + 1$ . More specifically,

$$\Lambda_0 = \begin{bmatrix} A - bK_0 & 0_{n \times \bar{\tau}n} \\ I_{\bar{\tau}n \times \bar{\tau}n} & 0_{\bar{\tau}n \times n} \end{bmatrix},$$

and

$$\Lambda_j = \begin{bmatrix} A & \underbrace{0 \cdots 0}_{j-1} & -bK_j & \underbrace{0 \cdots 0}_{\bar{\tau}-j} \\ I_{\bar{\tau}n \times \bar{\tau}n} & & 0_{\bar{\tau}n \times n} & \end{bmatrix},$$

$$j = 1, \dots, \bar{\tau}.$$

When the inverse of  $f(\cdot)$  cannot be obtained accurately, then

$$u(k) = \hat{g}(-K_{\tau_k} x(k - \tau_k)) \quad (13)$$

Then, the closed-loop system can be obtained as:

$$x(k+1) = Ax(k) + bf(\hat{g}(-K_{\tau_k} x(k - \tau_k))) \quad (14)$$

From the sector constraint (8), it can be obtained that there exists a real number  $\rho_k$  satisfying  $\rho_1 \leq \rho_k \leq \rho_2$  such that  $f(\hat{g}(\theta)) = \rho_k \theta$ . For this case, (14) is rewritten as:

$$x(k+1) = Ax(k) - \rho_k bK_{\tau_k} x(k - \tau_k) \quad (15)$$

The augmented closed-loop system can be obtained as:

$$\xi(k+1) = \Lambda_{\tau_k, \rho_k} \xi(k) \quad (16)$$

where  $\Lambda_{\tau_k, \rho_k}$  is obtained from  $\Lambda_{\tau_k}$  by replacing  $-bK_{\tau_k}$  with  $-\rho_k bK_{\tau_k}$ .

Recently, a switched Lyapunov function approach [24, 25] has been proposed to investigate the problem of stability and control synthesis for switched systems. This method only yields a sufficient stability condition, but the resulting condition is presented in terms of LMI and thus easy to test. This method is less conservative than those using a single quadratic Lyapunov function. In this paper, the switched Lyapunov function approach is used to derive a stability criterion for system (16).

**Theorem 1.** If there exists matrices  $P_i = P_i^T > 0$  and  $G_i$  ( $i \in \mathcal{I} = \{0, 1, 2, \dots, \bar{\tau}\}$ ) with appropriate dimensions such that the following LMIs hold for all  $(i, j) \in \mathcal{I} \times \mathcal{I}$ ,

$$\begin{bmatrix} -P_i & \Lambda_{i, \rho_1}^T G_i \\ G_i^T \Lambda_{i, \rho_1} & P_j - G_i - G_i^T \end{bmatrix} < 0 \quad (17)$$

$$\begin{bmatrix} -P_i & \Lambda_{i,\rho_2}^T G_i \\ G_i^T \Lambda_{i,\rho_2} & P_j - G_i - G_i^T \end{bmatrix} < 0 \quad (18)$$

where  $\Lambda_{i,\rho_1}$  and  $\Lambda_{i,\rho_2}$  are obtained from  $\Lambda_{i,\rho_k}$  by replacing  $\rho_k$  with  $\rho_1$  and  $\rho_2$ , respectively, then system (16) is asymptotically stable.

**Proof.** Define the following indicator function:

$$\alpha(k) = [\alpha_0(k) \alpha_1(k) \cdots \alpha_{\bar{\tau}}(k)]^T$$

with

$$\alpha_i(k) = \begin{cases} 1, & \text{when } \tau_k = i \\ 0, & \text{otherwise} \end{cases}$$

A switched Lyapunov function with the following structure is used to derive the stability condition for system (16)

$$V(k, \xi_k) = \xi_k^T P(\alpha(k)) \xi_k = \xi_k^T \left( \sum_{i=0}^{\bar{\tau}} \alpha_i(k) P_i \right) \xi_k \quad (19)$$

For all  $(i, j) \in \mathcal{I} \times \mathcal{I}$

$$\begin{aligned} \Delta V &= V(k+1, \xi_{k+1}) - V(k, \xi_k) \\ &= \xi_{k+1}^T P(\alpha(k+1)) \xi_{k+1} - \xi_k^T P(\alpha(k)) \xi_k \\ &= \xi_{k+1}^T P_j \xi_{k+1} - \xi_k^T P_i \xi_k \end{aligned} \quad (20)$$

It is easy to see that the following equation hold for all  $i \in \mathcal{I}$

$$0 = 2\xi_{k+1}^T G_i^T (-\xi_{k+1} + \Lambda_{i,\rho_k} \xi_k) \quad (21)$$

Adding both sides of (20) into both sides of (19), one can obtain

$$\begin{aligned} \Delta V &= \xi_{k+1}^T (P_j - G_i - G_i^T) \xi_{k+1} - \xi_k^T \\ &\quad \times P_i \xi_k + 2\xi_{k+1}^T G_i^T \Lambda_{i,\rho_k} \xi_k \end{aligned} \quad (22)$$

For any  $\rho_k \in [\rho_1, \rho_2]$ , there always exists  $0 \leq \lambda_k \leq 1$  such that  $\rho_k = \lambda_k \rho_1 + (1 - \lambda_k) \rho_2$ . Therefore,  $\Lambda_{i,\rho_k} = \lambda_k \Lambda_{i,\rho_1} + (1 - \lambda_k) \Lambda_{i,\rho_2}$ . One can obtained that

$$\begin{aligned} \Delta V &= \xi_{k+1}^T (P_j - G_i - G_i^T) \xi_{k+1} - \xi_k^T P_i \xi_k \\ &\quad + 2\lambda_k \xi_{k+1}^T G_i^T \Lambda_{i,\rho_1} \xi_k + 2(1 - \lambda_k) \xi_{k+1}^T \\ &\quad \times G_i^T \Lambda_{i,\rho_2} \xi_k \\ &= \lambda_k \begin{bmatrix} \xi_k \\ \xi_{k+1} \end{bmatrix}^T \end{aligned}$$

$$\begin{aligned} &\times \begin{bmatrix} -P_i & \Lambda_{i,\rho_1}^T G_i \\ G_i^T \Lambda_{i,\rho_1} & P_j - G_i - G_i^T \end{bmatrix} \\ &\times \begin{bmatrix} \xi_k \\ \xi_{k+1} \end{bmatrix} + (1 - \lambda_k) \begin{bmatrix} \xi_k \\ \xi_{k+1} \end{bmatrix}^T \\ &\times \begin{bmatrix} -P_i & \Lambda_{i,\rho_2}^T G_i \\ G_i^T \Lambda_{i,\rho_2} & P_j - G_i - G_i^T \end{bmatrix} \\ &\times \begin{bmatrix} \xi_k \\ \xi_{k+1} \end{bmatrix} \end{aligned} \quad (23)$$

Therefore, if (17) and (18) hold, then  $\Delta V < 0$  which guarantees system (16) is asymptotically stable. This completes the proof.  $\square$

For the case that the inverse of  $f(\cdot)$  is obtained accurately, the following corollary can be obtained on the basis of Theorem 1.

**Corollary 1.** If there exists matrices  $P_i = P_i^T > 0$  and  $G_i$  ( $i \in \mathcal{I}$ ) with appropriate dimensions such that the following LMI holds for all  $(i, j) \in \mathcal{I} \times \mathcal{I}$ ,

$$\begin{bmatrix} -P_i & \Lambda_i^T G_i \\ G_i^T \Lambda_i & P_j - G_i - G_i^T \end{bmatrix} < 0 \quad (24)$$

then system (12) is asymptotically stable.

#### IV. A NUMERICAL EXAMPLE

In this section, a numerical example is given to show the effectiveness of the proposed method.

Consider the following system with

$$A = \begin{bmatrix} 0.91 & 0.19 \\ -1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \quad c = [1 \ 0].$$

and the input nonlinearity is  $v = f(u) = u^2$ .

The above Hammerstein system is controlled over a network. It is assumed that the upper bound on the backward delay is  $\bar{\tau}_{sc} = 2$  and the upper bound on the forward delay is  $\bar{\tau}_{ca} = 2$ . Firstly, it is assumed that the inverse of  $f(\cdot)$  is obtained accurately. So, the input nonlinearity is totally removed by inverting it. For this case, if the linear part of the above Hammerstein system subject to the control scheme proposed in this paper is stable, then the nonlinear system will be stable. Choosing  $Q = 1$ ,  $R = 1$ ,  $N_u = 10$ , and  $N_p = 10$ , we compare our method with the LQR control method.

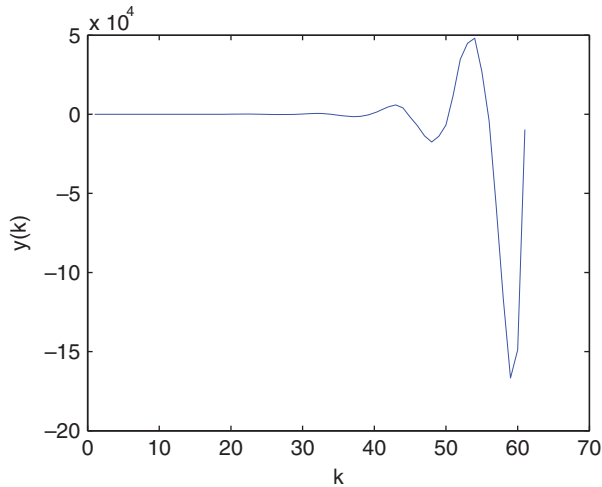


Fig. 1. Output of the system with LQR controller without delay compensation.

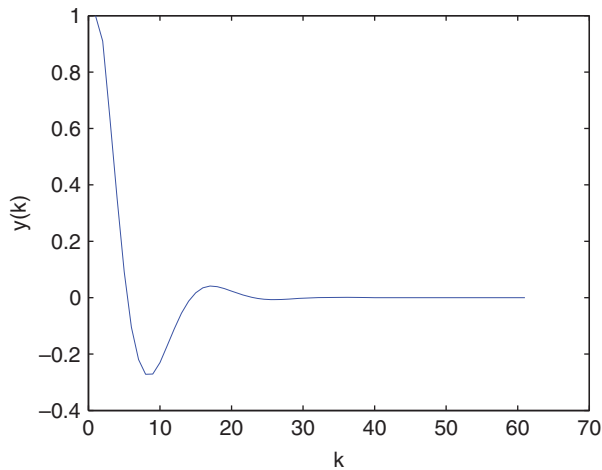


Fig. 2. Output of the system with delay compensation.

Simulation results with initial state  $x_0 = [1 \ 0]^T$  are given in Figs 1 and 2. It can be seen that the system is unstable when using the LQR control method without delay compensation, whereas the system is stable when using our networked predictive control method. It indicates that the method proposed in this paper is effective.

If the inverse of  $f(\cdot)$  is not obtained accurately and the practical inverse of  $f(\cdot)$  is  $\hat{g}(\cdot) = \rho\sqrt{v}$ . For the purpose of simulations,  $\rho$  is assumed to be a random number between  $[0.8 \ 1.2]$  or  $[0.9 \ 1.1]$  or  $[0.95 \ 1.05]$ . For these three cases, applying Theorem 1 in this paper, it can be proved that such a system is stable under our control scheme. Simulation results for

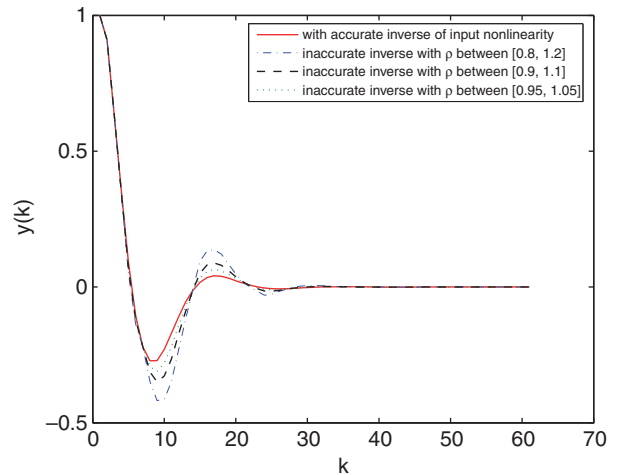


Fig. 3. Output of the system with accurate or inaccurate inverse of input nonlinearity.

these three cases with the same initial state  $x_0 = [1 \ 0]^T$  are given in Fig. 3. These results illustrate the validity of the theoretical analysis in this paper. From these results, it can be seen when  $f(\hat{g}(\cdot))$  is more approaching  $1(\cdot)$ , the response of the system is more approaching the case that the inverse of the input nonlinearity is accurate.

## V. CONCLUSIONS

In this paper, a networked predictive control method for Hammerstein systems has been proposed. This method can compensate for the random network-induced delay and data dropout. This method takes full advantage of the feature that predictive control method can generate the future control signals and the characteristic that data in a network are transmitted using a data packet. The input nonlinearity is compensated (or partially compensated) using its inverse. The stability criteria for the closed-loop system have been derived using the switched Lyapunov function approach. A numerical example has illustrated the effectiveness of the proposed method.

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