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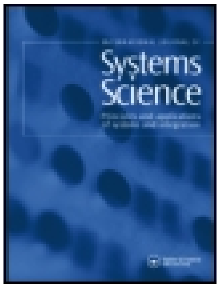

International Journal of Systems Science
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tsys20

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Available online: 26 Jul 2011

To cite this article: Hao Li, Lihua Dou & Zhong Su (2011): Adaptive nonsingular fast terminal sliding mode control for electromechanical actuator, International Journal of Systems Science, DOI:10.1080/00207721.2011.601348

To link to this article: http://dx.doi.org/10.1080/00207721.2011.601348

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International Journal of Systems Science
2011, 1–15, iFirst

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Adaptive nonsingular fast terminal sliding mode control for electromechanical actuator

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(*Received 1 July 2010; final version received 25 April 2011*)

An adaptive nonsingular fast terminal sliding mode control scheme consisting of an adaptive control term and a robust control term for electromechanical actuator is proposed in this article. The adaptive control term with an improved composite adaptive law can estimate the uncertain parameters and compensate for the modelled dynamical uncertainties. While the robust control term, which is based on a modified nonsingular fast terminal sliding mode control method with fast terminal sliding mode (TSM) reaching law, provides fast convergence of errors, and robustifies the design against unmodelled dynamics. Furthermore, the control method eliminates the singular problems in conventional TSM control. On the basis of the finite-time stability theory and the differential inequality principle, it is proved that the resulting closed-loop system is stable and the trajectory tracking error converges to zero in finite time. Finally the effectiveness of the proposed method is illustrated by simulation and experimental study.

Keywords: nonsingular fast terminal sliding mode control; composite adaptive law; electromechanical actuator; finite-time convergence

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1. Introduction

With the development of electronic technology, the electromechanical actuator (EMA) is now widely used in various applications. For example, it is utilised in aeroplanes, missiles, engine valves, injectors, brake systems and so on. As an important part of the entire system, EMA works like a converter or an amplifier that transfers the electrical signal to the mechanical movement. Consequently, the control of EMA, which is implemented to achieve faster and more precise regulations of position or velocity, has attracted much attention during the past few decades. However, the performance of EMA is influenced by uncertainties such as parameter variations, external disturbances and unmodelled dynamics. Therefore the control method should circumvent the uncertain problems to achieve better static and dynamic performance.

Sliding mode control (SMC), which provides invariance to uncertainties once the system dynamics are controlled on the sliding mode, is an efficient and effective robust approach to deal with control problems of uncertain systems. It has been widely used in practical systems, such as robot manipulators, DC–DC converters and motors (Utkin 1977, 1993; Utkin, Guldncr, and Shi 1999; Young, Utkin, and Ozguner 1999). There are two basic components in the SMC method: a stable sliding surface that ensures the desired dynamics, and a control effort that steers the system states to reach and stay on the sliding surface. Usually the sliding surface is a linear hyperplane of system states and only asymptotic stability is assured on the sliding manifold, which implies that system errors cannot converge to zero in finite time.

Terminal SMC (TSMC) is a variant scheme of SMC that can achieve finite-time stability (Bhat and Bernstein 1998, 2000). In Venkataraman and Gulati (1992), the attractor (Zak 1989) was adopted in the sliding surface, and TSMC for second-order SISO system was first presented. By employing the nonlinear sliding mode, TSMC offers a finite-time error convergence. Inspired by this idea, researchers developed TSMC approaches with high-order systems (Yu and Man 1996), MIMO linear systems (Man and Yu 1997) and uncertain dynamic systems (Wu, Yu, and Man 1998). Nevertheless, TSMC cannot deliver the same convergence performance while the system states are far away from the equilibrium point. To overcome this problem, Yu and Man (2002) presented the fast TSMC (FTSMC) method that can achieve fast finite-time convergence when the states are either far away from or near the equilibrium point. However, singularity occurs in both TSMC and FTSMC, and this issue was

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ISSN 0020–7721 print/ISSN 1464–5319 online
© 2011 Taylor & Francis
DOI: 10.1080/00207721.2011.601348
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addressed explicitly in literature (Feng, Yu, and Man 2002; Yu, Yu, Shirinzadeh, and Man 2005; Jin, Lee, Chang, and Choi 2009), where global nonsingular TSMC (NTSMC) methods with the same convergence properties as those of TSMC for uncertain systems were proposed. To achieve fast finite-time convergence for NTSMC, Yu, Du, Yu, and Xu (2008) introduced the nonsingular FTSMC (NFTSMC) for a class of n-order systems, but a high gain control effort was adopted to ensure the stability of the close-loop system.

The aforementioned control approaches, i.e. SMC, TSMC, FTSMC and their nonsingular forms, are robust control methods that do not have the ability to 'learn' in the control process, and result in a conservative design. Furthermore, the stability of the system is achieved at the cost of performance (Song, Longman, and Mukherjee 1999). While adaptive SMC (ASMC), with the integration of adaptive control and SMC, may ride this disadvantage and has been widely investigated in recent years (Man, Mike, and Yu 1999; Keleher and Stonier 2002; Zhao, Li, Gao, and Zhu 2009). Usually only the tracking error is used in the adaptive law in ASMC, and this brings about slow parameter convergence and large transient tracking error. In Barambones and Etxebarria (2001, 2002), a modified TSMC with the composite adaptive law (Slotine and Li 1989) was presented, where the parameters were estimated by both the tracking error and the prediction error. Faster parameter convergence and smaller tracking errors were achieved, and true parameter estimates could be acquired if the persistent excitation (PE) condition was satisfied. To ensure the boundedness of all signals, a nonlinear filtered error signal which was switched to zero (Barambones and Etxebarria 2001, 2002) when the trajectory error equalled zero was introduced in TSMC. However, the control effort may become awfully large while the trajectory error is close to zero, and only semi-global nonsingularity is obtained.

In this article the above-mentioned problems are addressed. An adaptive NFTSMC (ANFTSMC) approach for EMA is proposed. The control scheme comprises an adaptive control term and a robust control term. The adaptive control term uses a novel composite adaptive law where the integration of filtered system states are employed to estimate the parameters, and then compensates for the modelled dynamic uncertainties. While the robust term, which is based on a modified NFTSMC with the fast TSM reaching law, provides fast finite-time convergence of errors either far away from or near the equilibrium point. In addition it robustifies the design against unmodelled dynamics with a small switching gain duo to the parameter adaption. Consequently, the control effort is less conservative than that of NFTSMC (Yu et al. 2008). The adaptive law in ANFTSMC can acquire accurate parameter estimates under a weaker condition than that proposed by Barambones and Etxebarria (2001, 2002). Furthermore, the semi-global singular issue in Barambones and Etxebarria (2001, 2002) and Zhao, Li, and Gao (2009) is eliminated as the NFTSMC approach is adopted, and global nonsingularity is achieved. The main contributions of this article are as follows: (1) a novel architecture of ANFTSMC with the combination of composite adaptive law and NFTSMC that assures fast finite-time error convergence is provided. (2) The stability and error convergence analysis of the proposed ANFTSMC are given.

This article is organised as follows. Model of EMA is described and problem formulation is given in Section 2. In Section 3 the control design is introduced. Stability and error convergence analysis are presented in Section 4. Simulation and experimental study are given in Section 5 and some conclusions are drawn in Section 6.

2. Models and problem formulation

2.1. *Dynamic models of EMA*

EMA with a DC motor driving a gearbox mechanism will be concerned. Generally the current dynamics of the motor is neglected due to the much faster electric response in comparison to the mechanical dynamics. The model of the actuator can be described as follows (Ilyas 2006),

$$J\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2}+f\frac{\mathrm{d}\delta}{\mathrm{d}t}+M_L+\Delta=K_1u, \tag{1}$$

where δ is the output angle of the gearbox shaft, J, f, M_L, K_1 and Δ are equivalent parameters relative to the actuator shaft: total moment of inertia, total damping coefficient, load torque, equivalent electrical-mechanical energy conversion constant and unmodelled dynamics of the actuator.

Usually in servo systems the load is figured as an unknown torque, but in some cases the position dependent torque which can be modelled as a spring load torque is met (such as the hinge moment of the aerocraft). Thus, the load torque of EMA can be expressed as (Wu and Fei 2005),

$$M_L=M_j^\delta\delta+M_C, \tag{2}$$

where M_j^δ is the coefficient of spring load torque and M_C is the constant load torque.

Regarding the output angle δ as the system output y, and defining the output angle and angular velocity of the actuator as the state variables,

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i.e., $x = [x_1, x_2]^T = [\delta, \dot{\delta}]^T$, then the entire system can be expressed as

$$\begin{cases} \dot{x}_1 = x_2, \\ \theta_1 \dot{x}_2 = u - \theta_2 x_1 - \theta_3 x_2 - \theta_4 - \Delta', \\ y = x_1, \end{cases} \tag{3}$$

where $\Delta' = \Delta / K_1(\mathrm{V})$, and $\theta_i (i = 1, 2, 3, 4)$ are given as follows:

$$\begin{aligned} \theta_1 &= \frac{J}{K_1}\left(\mathrm{V} \cdot (\mathrm{rad} \cdot \mathrm{s}^{-2})^{-1}\right), \\ \theta_2 &= \frac{M_j^\delta}{K_1}\left(\mathrm{V} \cdot \mathrm{rad}^{-1}\right), \\ \theta_3 &= \frac{f}{K_1}\left(\mathrm{V} \cdot (\mathrm{rad} \cdot \mathrm{s}^{-1})^{-1}\right), \\ \theta_4 &= \frac{M_C}{K_1}(\mathrm{V}). \end{aligned} \tag{4}$$

2.2. *Assumptions and problem formulation*

For simplicity, the following notations will be used: $\bullet_i$ for the ith component of the vector $\bullet$, $\hat{\bullet}$ for the estimate of $\bullet$, $\bullet_{\min}$ for the minimum value of $\bullet$ and $\bullet_{\max}$ for the maximum value of $\bullet$. $\| \bullet \|$ is the Euclidean norm of $\bullet$. $\lambda_{\min}(*)$ and $\lambda_{\max}(*)$ are the minimum eigenvalue and maximum eigenvalue of the matrix $*$, respectively. The operation $\geq$ (or $\leq$) for two vectors is performed in terms of the corresponding elements.

In general, the parameters of the model cannot be accurately determined, but we assume that the uncertain parameters lie in some previously known intervals, as shown in Assumptions 1 and 2. In addition, Assumption 3 is for the desired trajectories (Zhang, Chen, and Li 2010).

Assumption 1: *$\theta_{\min} \leqslant \theta \leqslant \theta_{\max}$, $\theta_{\min}$ and $\theta_{\max}$ are known with $\theta_{\min} = [\theta_{1,\min}, \ldots, \theta_{4,\min}]^{\mathrm{T}}$, $\theta_{\max} = [\theta_{1,\max}, \ldots, \theta_{4,\max}]^{\mathrm{T}}$. Moreover, $\theta_{1,\min} > 0$, which conforms to the physical point of view.*

Assumption 2: *The unmodelled dynamics is bounded, i.e., $\|\Delta'\| \leq \varsigma$, where $\varsigma > 0$ is known.*

Assumption 3: *The desired trajectory x_d is continuous, and its first-order derivative $\dot{x}_d$ and second-order derivative $\ddot{x}_d$, are bounded and available.*

Consider model (3) which has unknown parameters, disturbances and unmodelled dynamics, the control problem of this article can be formulated as follows. Given the desired motion trajectory x_d, the object is to synthesise a control action u such that the system tracking error $e_1 = x_1 - x_d$ converges to zero, while maintaining all signals in the system bounded.

3. Controller design

3.1. *Control law design*

In this section the nonsingular fast terminal sliding mode (NFTSM) for EMA is first introduced. Then the control law comprising an adaptive control term and a robust control term is designed.

NFTSM (Yu et al. 2008) for model (3) can be described as

$$\begin{cases} \sigma_1 = e_1, \\ \sigma_2 = \sigma_1 + \dfrac{\beta}{2 - \gamma} |\dot{\sigma}_1 + c\sigma_1|^{2-\gamma} \operatorname{sign}(\dot{\sigma}_1 + c\sigma_1), \end{cases} \tag{5}$$

where $\beta > 0$, $c > 0$, $\gamma = z_1/z_2$, $0 < z_1 < z_2$, z_1 and z_2 are odd integers. β, c, γ are parameters to be designed. The signum function sign($\star$) for the scalar $\star$ is defined as

$$\operatorname{sign}(\star) = \begin{cases} 1, & \star > 0, \\ 0, & \star = 0, \\ -1, & \star < 0. \end{cases} \tag{6}$$

The first derivative of σ_2 is as follows (Yu et al. 2005, 2008):

$$\dot{\sigma}_2 = \dot{\sigma}_1 + \beta |\dot{\sigma}_1 + c\sigma_1|^{1-\gamma} (\ddot{\sigma}_1 + c\dot{\sigma}_1). \tag{7}$$

Remark 3.1: In Barambones and Etxebarria (2001, 2002) and Zhao et al. (2009), a nonlinear filtered error named as e_r is used in sliding mode design. e_r is switched to zero (Barambones and Etxebarria 2001, 2002) or a constant value (Zhao et al. 2009) when the trajectory error e equals zero, and all the internal signals are bounded when $e = 0$. However, e_r may become awfully large as $e \to 0$, and it does not achieve global nonsingularity. The details can be seen in Appendix A. In (7), as $\gamma < 1$, $1 - \gamma > 0$, then $\dot{\sigma}_2$ and σ_2 are bounded when $(\dot{\sigma}_1 + c\sigma_1) \to 0$. Thus, the singular problems in classic TSMC and FTSMC are avoided and global nonsingularity can be achieved (Yu et al. 2008).

Noting (3) and (7), we obtain the following dynamics.

$$\begin{aligned} \theta_1 \dot{\sigma}_2 &= \theta_1 \dot{\sigma}_1 + \beta |\dot{\sigma}_1 + c\sigma_1|^{1-\gamma} (\theta_1 \ddot{\sigma}_1 + \theta_1 c \dot{\sigma}_1) \\ &= \theta_1 \dot{\sigma}_1 + \beta |\dot{\sigma}_1 + c\sigma_1|^{1-\gamma} (u - \theta_2 x_1 - \theta_3 x_2 - \theta_4 \\ &\quad - \Delta' - \theta_1 \ddot{x}_d + \theta_1 c \dot{\sigma}_1). \end{aligned} \tag{8}$$

Then the control law is as follows

$$u = u_a + u_s, \tag{9}$$

where u_a denotes the adaptive control term in (10) and u_s is the robust control term in (13). Let:

$$u_a = \hat{\theta}_1 [\ddot{x}_d - c\dot{\sigma}_1 - \varphi(\sigma_1)] + \hat{\theta}_2 x_1 + \hat{\theta}_3 x_2 + \hat{\theta}_4 = \hat{\theta}^{\mathrm{T}} \varphi_1, \tag{10}$$

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where $\hat{\theta} = [\hat{\theta}_1, \ldots, \hat{\theta}_4]^{\mathrm{T}}$, $\hat{\theta}_i$ is the estimate of θ_i, $i = 1, 2, 3, 4$. φ_1 and $\varphi(\sigma_1)$ are given by

$$\varphi_1 = [\ddot{x}_d - c\dot{\sigma}_1 - \varphi(\sigma_1), x_1, x_2, 1]^{\mathrm{T}}, \tag{11}$$

$$\varphi(\sigma_1) = \frac{1}{\beta}|\dot{\sigma}_1 + c\sigma_1|^{\gamma}\mathrm{sign}(\dot{\sigma}_1 + c\sigma_1) + \frac{c}{2-\gamma}(\dot{\sigma}_1 + c\sigma_1). \tag{12}$$

The robust control term contains two parts

$$\begin{cases} u_s = u_{s,1} + u_{s2}, \\ u_{s,1} = -k_1\sigma_2 - k_2|\sigma_2|^r \,\mathrm{sign}(\sigma_2), \\ u_{s,2} = -k_3 \,\mathrm{sign}(\sigma_2), \end{cases} \tag{13}$$

where $k_1 > 0$, $k_2 > 0$, $k_3 > \varsigma$, $r = z_3/z_4$, $0 < z_3 < z_4$, z_3 and z_4 are odd integers. k_1, k_2, k_3 and r are parameters to be designed.

Remark 3.2: The first part of the robust control term, $u_{s,1}$, is used to construct a fast TSM type reaching law (Yu et al. 2005) that can ensure high convergence speed as the system states are either far away from or near the sliding manifold. The second part of the robust control term, $u_{s,2}$, is adopted to robustify the system against unmodelled dynamics.

3.2. *Adaptive law design*

Substituting (9)–(13) into (8), we get

$$\begin{aligned} \theta_1\dot{\sigma}_2 = \theta_1\dot{\sigma}_1 - \hat{\theta}_1\beta|\dot{\sigma}_1 + c\sigma_1|^{1-\gamma}\varphi(\sigma_1) \\ + \beta|\dot{\sigma}_1 + c\sigma_1|^{1-\gamma}[\tilde{\theta}_1(\ddot{x}_d - c\dot{\sigma}_1) + \tilde{\theta}_2 x_1 \\ + \tilde{\theta}_3 x_2 + \tilde{\theta}_4 + u_s - \Delta'], \end{aligned} \tag{14}$$

where $\tilde{\theta}_i$ is defined as $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $i = 1, 2, 3, 4$.

Noting (5) and (12), we can obtain that

$$\begin{aligned} &\beta|\dot{\sigma}_1 + c\sigma_1|^{1-\gamma}\varphi(\sigma_1) \\ &= \beta|\dot{\sigma}_1 + c\sigma_1|^{1-\gamma} \\ &\quad\times\left[\frac{1}{\beta}|\dot{\sigma}_1 + c\sigma_1|^{\gamma}\mathrm{sign}(\dot{\sigma}_1 + c\sigma_1) + \frac{c}{2-\gamma}(\dot{\sigma}_1 + c\sigma_1)\right] \\ &= \dot{\sigma}_1 + c\sigma_1 + \frac{c\beta}{2-\gamma}|\dot{\sigma}_1 + c\sigma_1|^{2-\gamma}\mathrm{sign}(\dot{\sigma}_1 + c\sigma_1) \\ &= \dot{\sigma}_1 + c\sigma_2. \end{aligned} \tag{15}$$

Let $\beta' = \beta|\dot{\sigma}_1 + c\sigma_1|^{1-\gamma}$ and substituting (15) into (14) results in

$$\begin{aligned} \theta_1\dot{\sigma}_2 &= \tilde{\theta}_1[-(\dot{\sigma}_1 + c\sigma_2) + \beta'(\ddot{x}_d - c\dot{\sigma}_1)] + \tilde{\theta}_2\beta' x_1 + \tilde{\theta}_3\beta' x_2 \\ &\quad + \tilde{\theta}_4\beta' - c\theta_1\sigma_2 + \beta'(u_s - \Delta') \\ &= -\tilde{\theta}^{\mathrm{T}}\varphi_2 - c\theta_1\sigma_2 + \beta'(u_s - \Delta'), \end{aligned} \tag{16}$$

where

$$\varphi_2 = [\dot{\sigma}_1 + c\sigma_2 - \beta'(\ddot{x}_d - c\dot{\sigma}_1),\ -\beta' x_1,\ -\beta' x_2,\ -\beta']^{\mathrm{T}}.$$

The adaptive law for $\hat{\theta}$ can be chosen as

$$\dot{\hat{\theta}} = \mathrm{proj}_{\hat{\theta}}\left(\Gamma_1\varphi_2\sigma_2 - \Gamma_1\Gamma_2 e_f - \Gamma_1\Gamma_3\mathrm{sig}(e_f)^r\right), \tag{17}$$

where $\Gamma_i = \mathrm{diag}(\Gamma_{i1}, \Gamma_{i2}, \Gamma_{i3}, \Gamma_{i4})$, $\Gamma_{ij} > 0$, $i = 1, 2, 3$; $j = 1, 2, 3, 4$. The projection operator is defined as $\mathrm{proj}_{\hat{\theta}}(\bullet) = [\mathrm{proj}_{\hat{\theta}_1}(\bullet_1), \ldots, \mathrm{proj}_{\hat{\theta}_4}(\bullet_4)]^{\mathrm{T}}$ with (Zhang, Chen, and Li 2009; Zhang et al. 2010)

$$\mathrm{proj}_{\hat{\theta}_i}(\bullet_i) = \begin{cases} 0, & \hat{\theta}_i = \theta_{i,\max} \text{ and } \bullet_i > 0, \\ 0, & \hat{\theta}_i = \theta_{i,\min} \text{ and } \bullet_i < 0, \\ \bullet_i, & \text{others}. \end{cases} \tag{18}$$

The prediction error e_f in (17) is given by Zhang et al. (2009)

$$e_f = P\hat{\theta} - Q, \tag{19}$$

and $\mathrm{sig}(e_f)^r$ is defined as

$$\mathrm{sig}(e_f)^r = [|e_{f,1}|^r\mathrm{sign}(e_{f,1}), \ldots, |e_{f,4}|^r\mathrm{sign}(e_{f,4})]^{\mathrm{T}}. \tag{20}$$

While the matrix P and Q are as follows:

$$\begin{aligned} P &= \int_0^t [\varphi_f\ell(\tau)][\varphi_f\ell(\tau)]^{\mathrm{T}}\mathrm{d}\tau, \\ Q &= \int_0^t [\varphi_f\ell(\tau)][u_f\ell(\tau)]\mathrm{d}\tau, \end{aligned} \tag{21}$$

where φ_f and u_f are filtered signals related to the system states and control input u, i.e.

$$\begin{cases} \dot{\varphi}_{f,2} + \kappa_f\varphi_{f,2} = \kappa_f x_1, \\ \dot{\varphi}_{f,3} + \kappa_f\varphi_{f,3} = \kappa_f x_2, \\ \dot{\varphi}_{f,4} + \kappa_f\varphi_{f,4} = \kappa_f, \\ \dot{u}_f + \kappa_f u_f = \kappa_f u, \end{cases} \tag{22}$$

while $\varphi_{f,1} = \kappa_f(x_2 - \varphi_{f,3})$, and $\varphi_f = [\varphi_{f,1}, \varphi_{f,2}, \varphi_{f,3}, \varphi_{f,4}]^{\mathrm{T}}$. κ_f is a large positive number such that the filter outputs can track their corresponding inputs closely.

$\ell(\tau)$ is given by

$$\ell(\tau) = \exp(-\varepsilon_1|\tau|), \tag{23}$$

where ε_1 is a positive parameter to be designed.

Remark 3.3: According to Yu et al. (2008), the control effort of NFTSMC for (3) is $u = k\,\mathrm{sign}(\sigma_2)$. If $k > \|\theta^{\mathrm{T}}\| \cdot \|\varphi_1\| + \varsigma$, then the system is stable and the finite-time error convergence can be achieved. For uncertain parameters with known bounds k is chosen as $k > \|\theta^{\mathrm{T}}\|_{\max} \cdot \|\varphi_1\| + \varsigma$, where $\|\theta^{\mathrm{T}}\|_{\max}$ is the maximum value of $\|\theta^{\mathrm{T}}\|$, and $\|\theta^{\mathrm{T}}\|_{\max} = (\sum_{i=1}^{4}|\theta_i|_{\max}^2)^{1/2}$, $|\theta_i|_{\max} = \max\{|\theta_{i,\max}|, |\theta_{i,\min}|\}$, $i = 1, 2, 3, 4$. However, the system's parameters may rarely bear the bounds in practical, and $\|\theta^T\|_{\max}\cdot\|\varphi_1\|$ is far larger than $\theta^T\varphi_1$. While in ANFTSMC, $u_{s,1}$ decays to zero when the system states reach the sliding mode σ_2, and a small switching gain can be used in $u_{s,2}$ (in the presented method $k_3>\varsigma$) to ensure the system's stability (the stability analysis is proposed in Section 4) due to parameter adaption. Thus, the control effort in ANFTSMC is less conservative than that in NFTSMC.

Remark 3.4: Owing to the construction of the sliding mode σ_2 in (5), the action of $\dot\sigma_1$ shown in (8) should be balanced in view of system stability. It is not a special issue in NFTSMC for a high gain control effort is employed to stabilise the system. If k is large enough, the action of $\dot\sigma_1$ can be counteracted when $\dot\sigma_1+c\sigma_1\neq 0$. However, an additional item that can offset the action of $\dot\sigma_1$ is required in ANFTSMC to design the adaptive control term, where a linearly parameter dependent term in the control effort is necessary in terms of parameter adaption. Intuitively an item in the form of $\beta|\dot\sigma_1+c\sigma_1|^{\gamma-1}\dot\sigma_1$ may be used in the control law. But it makes the control effort unbounded when $\dot\sigma_1+c\sigma_1\to 0$ and $\dot\sigma_1\neq 0$, for $\beta|\dot\sigma_1+c\sigma_1|^{\gamma-1}\dot\sigma_1\to\infty$ in this case as $\gamma<1$, i.e. a new singular problem may be brought. Here, the item $\varphi(\sigma_1)$ is introduced in the adaptive control term. It can counteract the influence of $\dot\sigma_1$ as shown in (14)–(16). While $\dot\sigma_1+c\sigma_1\to 0$, $\varphi(\sigma_1)\to 0$, which ensures the boundedness of the control effort.

Remark 3.5: Noting (3), the control input u can also be expressed as

$$u=\theta_1\dot x_2+\theta_2x_1+\theta_3x_2+\theta_4+\Delta'=\varphi_r^{\mathrm T}\theta+\Delta', \tag{24}$$

where $\varphi_r=[\dot x_2,x_1,x_2,1]^{\mathrm T}$. Substituting (24) into (22), u_f can be rewritten as

$$u_f=\varphi_f^{\mathrm T}\theta+\varphi_{f,\Delta'}, \tag{25}$$

where $\varphi_{f,\Delta'}$ is the filtered output of Δ', i.e., $\dot\varphi_{f,\Delta'}+\kappa_f\varphi_{f,\Delta'}=\kappa_f\Delta'$. Due to the boundedness of Δ', $\varphi_{f,\Delta'}$ is bounded. Assume that $\varphi_{f,\Delta'}=0$, then we can get (Zhang et al. 2010)

$$u_f=\varphi_f^{\mathrm T}\theta. \tag{26}$$

From (19) and (21), we have (Zhang et al. 2009)

$$\begin{aligned}e_f=P\hat\theta-Q&=\int_0^t[\varphi_f\ell(\tau)][\varphi_f\ell(\tau)]^{\mathrm T}\mathrm d\tau\,\hat\theta\\&\quad-\int_0^t[\varphi_f\ell(\tau)][u_f\ell(\tau)]\mathrm d\tau\\&=\int_0^t[\varphi_f\ell(\tau)][\varphi_f\ell(\tau)]^{\mathrm T}\mathrm d\tau\,\hat\theta\\&\quad-\int_0^t[\varphi_f\ell(\tau)][\varphi_f\ell(\tau)]^{\mathrm T}\mathrm d\tau\,\theta\\&=\int_0^t[\varphi_f\ell(\tau)][\varphi_f\ell(\tau)]^{\mathrm T}\mathrm d\tau\,\tilde\theta=P\tilde\theta.\end{aligned} \tag{27}$$

Remark 3.6: In Zhang et al. (2009), only φ_f is used to construct the matrix P and Q. As P is non-negative definite, it may become infinite when $t\to\infty$. In order to keep P bounded φ_f is reset to 0 after some time in Zhang et al. (2010), where the invertibility of P needs to be checked online. Here, a fading term $\ell(\tau)$ given in (23) is employed. As $\tau\to\infty$, $\ell(\tau)$ converges to zero exponentially, this can ensure the boundedness of P for bounded φ_f.

4. Stability and error convergence analysis

Before the stability analysis, some properties of the discontinuous projection mapping used in (17) are given as follows.

P1 (Ioannou and Sun 1996):

$$\tilde\theta[\Gamma^{-1}\mathrm{proj}_{\hat\theta}(\Gamma\nu)]\leqslant 0\quad\forall\nu\in R. \tag{28}$$

P2 (Zhang et al. 2009):

$$\tilde\theta[\Gamma^{-1}\mathrm{proj}_{\hat\theta}(\Gamma\nu-\Gamma\nu')-\nu]\leqslant-\tilde\theta\nu'\quad\forall\nu\in R,\ \forall\nu'\in R. \tag{29}$$

P3 (Ioannou and Sun 1996): If $\hat\theta(t)\in\Omega_\theta\triangleq\{\hat\theta:\theta_{\min}\leqslant\hat\theta\leqslant\theta_{\max}\}$ and the adaptive law is $\dot{\hat\theta}(t)=\mathrm{proj}(\Gamma\nu)$ in the time interval $[t',t'']$, then

$$\hat\theta(t)\in\Omega_\theta\quad\forall t\in[t',t'']. \tag{30}$$

Theorem 4.1: *Suppose that the control law in* (9)–(13) *with the adaptive law in* (17)–(23) *is applied to the plant* (3), *then the controller guarantees that*:

(i) *The closed-loop system is globally stable.*
(ii) *If there exists a time $T_0(T_0>0)$ that $P(T_0)$ is positive definite, the system trajectory error converges to zero in fast finite-time form.*

Proof: Define the following Lyapunov function candidate

$$V=V_1+V_2, \tag{31}$$

$$V_1=\frac{1}{2}\theta_1\sigma_2^2, \tag{32}$$

$$V_2=\frac{1}{2}\tilde\theta^{\mathrm T}\Gamma_1^{-1}\tilde\theta. \tag{33}$$

The derivative of V satisfies

$$\dot V=\dot V_1+\dot V_2=\sigma_2\theta_1\dot\sigma_2+\tilde\theta^{\mathrm T}\Gamma_1^{-1}\dot{\tilde\theta}. \tag{34}$$

We first prove (i).

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Substituting (16), (17) into (34) and noting **P2**, we can get

$$\begin{aligned}\dot{V} &= \sigma_2[-\tilde{\theta}^{\mathrm{T}}\varphi_2 - c\theta_1\sigma_2 + \beta'(u_s - \Delta')] + \tilde{\theta}^{\mathrm{T}}\Gamma_1^{-1}\dot{\hat{\theta}} \\ &= -c\theta_1\sigma_2^2 + \beta'\sigma_2[-k_1\sigma_2 - k_2|\sigma_2|^r \mathrm{sign}(\sigma_2) \\ &\quad - k_3\,\mathrm{sign}(\sigma_2) - \Delta'] \\ &\quad - \tilde{\theta}^{\mathrm{T}}\varphi_2\sigma_2 + \tilde{\theta}^{\mathrm{T}}\Gamma_1^{-1}\mathrm{proj}_{\hat{\theta}}\big(\Gamma_1\varphi_2\sigma_2 - \Gamma_1\Gamma_2 e_f \\ &\quad - \Gamma_1\Gamma_3\,\mathrm{sig}(e_f)^r\big) \\ &\leqslant -c\theta_1\sigma_2^2 - \beta'(k_1\sigma_2^2 + k_2|\sigma_2|^{1+r}) - \beta'(k_3|\sigma_2| + \Delta'\sigma_2) \\ &\quad - \tilde{\theta}^{\mathrm{T}}\Gamma_2 e_f - \tilde{\theta}^{\mathrm{T}}\Gamma_3\,\mathrm{sig}(e_f)^r. \end{aligned} \tag{35}$$

From (20), we have

$$\mathrm{sig}(e_f)^r = E_d e_f, \tag{36}$$

where $E_d = \mathrm{diag}(e_{d,1}, e_{d,2}, e_{d,3}, e_{d,4})$, and $e_{d,i}$ $(i = 1, 2, 3, 4)$ is defined as

$$e_{d,i} = \begin{cases} |e_{f,i}|^{r-1}, & e_{r,i} \neq 0, \\ 0, & e_{r,i} = 0. \end{cases} \tag{37}$$

Noting (27), then

$$\tilde{\theta}^{\mathrm{T}}\Gamma_2 e_f + \tilde{\theta}^{\mathrm{T}}\Gamma_3\,\mathrm{sig}(e_f)^r = \tilde{\theta}^{\mathrm{T}}\Gamma_2 P\tilde{\theta} + \tilde{\theta}^{\mathrm{T}}\Gamma_3 E_d P\tilde{\theta} = \tilde{\theta}^{\mathrm{T}}\Xi\tilde{\theta}, \tag{38}$$

where $\Xi = \Gamma_2 P + \Gamma_3 E_d P$. As Γ_2 and Γ_3 are symmetric positive definite matrices, while E_d and P are non-negative definite, then Ξ is non-negative definite, and $\tilde{\theta}^{\mathrm{T}}\Xi\tilde{\theta} \geqslant 0$, i.e.

$$\tilde{\theta}^{\mathrm{T}}\Gamma_2 e_f + \tilde{\theta}^{\mathrm{T}}\Gamma_3\,\mathrm{sig}(e_f)^r \geqslant 0. \tag{39}$$

While $k_3 > \varsigma$, it is verified that

$$\begin{aligned}\beta'(k_3|\sigma_2| + \Delta'\sigma_2) &\geqslant \beta'(k_3|\sigma_2| - |\Delta'||\sigma_2|) \\ &\geqslant \beta'(k_3 - |\Delta'|)|\sigma_2| \geqslant 0. \end{aligned} \tag{40}$$

Thus

$$\dot{V} \leqslant -c\theta_1\sigma_2^2 - \beta'(k_1\sigma_2^2 + k_2|\sigma_2|^{1+r}). \tag{41}$$

When $\sigma_2 \neq 0$, $\dot{V} < 0$. The condition for Lyapunov stability is satisfied. Noting Assumption 1 and **P3**, it holds that $|\tilde{\theta}_i| \leqslant |\theta_{i,\max} - \theta_{i,\min}|$, $i = 1, 2, 3, 4$. So V_2 is bounded. Moreover, $V_1 \to \infty$ as $\sigma_2 \to \infty$, thus $V \to \infty$ as $\sigma_2 \to \infty$, i.e., V is radially unbounded. This completes the proof of (i).

Then we prove (ii) in three cases.

Case 1: $\sigma_2 \neq 0$ and $\dot{\sigma}_1 + c\sigma_1 \neq 0$.

When $\sigma_2 \neq 0$ and $\dot{\sigma}_1 + c\sigma_1 \neq 0$, we get the following inequality by (35) and (40).

$$\begin{aligned}\dot{V} &\leqslant -c\theta_1\sigma_2^2 - \beta'(k_1\sigma_2^2 + k_2|\sigma_2|^{1+r}) \\ &\quad - \tilde{\theta}^{\mathrm{T}}\Gamma_2 e_f - \tilde{\theta}^{\mathrm{T}}\Gamma_3\,\mathrm{sig}(e_f)^r. \end{aligned} \tag{42}$$

The terms related to σ_2 on the right hand of (42) satisfy

$$\begin{aligned} &-c\theta_1\sigma_2^2 - \beta'(k_1\sigma_2^2 + k_2|\sigma_2|^{1+r}) \\ &= -2(c + \beta' k_1/\theta_1)V_1 - (2/\theta_1)^{\frac{1+r}{2}}\beta' k_2 V_1^{\frac{1+r}{2}} \\ &\leqslant -2(c + \beta' k_1/\theta_{1,\max})V_1 - (2/\theta_{1,\max})^{\frac{1+r}{2}}\beta' k_2 V_1^{\frac{1+r}{2}} \\ &= -k_1' V_1 - k_2' V_1^{\frac{1+r}{2}}, \end{aligned} \tag{43}$$

where $k_1' = 2(c + \beta' k_1/\theta_{1,\max})$, $k_2' = (2/\theta_{1,\max})^{\frac{1+r}{2}}\beta' k_2$.

If $P(T_0)$ is positive definite at sometime T_0, then $\lambda_{\min}(P(T_0)) > 0$, for $t \geqslant T_0$ we have (Zhang et al. 2009)

$$\begin{aligned} P(t) &= \int_0^t [\varphi_f\ell(t)][\varphi_f\ell(t)]^{\mathrm{T}}\mathrm{d}\tau \\ &= \int_0^{T_0} [\varphi_f\ell(t)][\varphi_f\ell(t)]^{\mathrm{T}}\mathrm{d}\tau + \int_{T_0}^t [\varphi_f\ell(t)][\varphi_f\ell(t)]^{\mathrm{T}}\mathrm{d}\tau \\ &= P(T_0) + \int_{T_0}^t [\varphi_f\ell(t)][\varphi_f\ell(t)]^{\mathrm{T}}\mathrm{d}\tau, \end{aligned} \tag{44}$$

thus $\forall t \geqslant T_0$, $P(t)$ is invertible if $P(T_0)$ is positive definite, and $\lambda_{\min}(P(t)) \geqslant \lambda_{\min}(P(T_0)) > 0$.

Then the terms related to e_f on the right hand of (42) satisfy

$$\begin{aligned} &-\tilde{\theta}^{\mathrm{T}}\Gamma_2 e_f - \tilde{\theta}^{\mathrm{T}}\Gamma_3\,\mathrm{sig}(e_f)^r \\ &= -\tilde{\theta}^{\mathrm{T}}P^{\mathrm{T}}(P^{\mathrm{T}})^{-1}\Gamma_2 e_f - \tilde{\theta}^{\mathrm{T}}P^{\mathrm{T}}(P^{\mathrm{T}})^{-1}\Gamma_3\,\mathrm{sig}(e_f)^r \\ &\leqslant -\frac{\lambda_{\min}(\Gamma_2)}{\lambda_{\max}(P^{\mathrm{T}})}e_f^{\mathrm{T}}e_f - \frac{\lambda_{\min}(\Gamma_3)}{\lambda_{\max}(P^{\mathrm{T}})}e_f^{\mathrm{T}}\,\mathrm{sig}(e_f)^r. \end{aligned} \tag{45}$$

According to Lemma B2, the following inequality holds.

$$\begin{aligned} e_f^{\mathrm{T}}\,\mathrm{sig}(e_f)^r &= \sum_{j=1}^4 |e_{f,j}|^{(1+r)} \geqslant \left(\sum_{j=1}^4 |e_{f,j}|^2\right)^{(1+r)/2} \\ &= (e_f^{\mathrm{T}}e_f)^{(1+r)/2}. \end{aligned} \tag{46}$$

While for $\Gamma_i = \mathrm{diag}(\Gamma_{i1}, \Gamma_{i2}, \Gamma_{i3}, \Gamma_{i4})$, we have

$$\begin{aligned} V_2 &= \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta} \leqslant \frac{1}{2}\lambda_{\max}(\Gamma_1^{-1})\tilde{\theta}^{\mathrm{T}}\tilde{\theta} \\ &= \frac{1}{2}\lambda_{\max}(\Gamma_1^{-1})\tilde{\theta}^{\mathrm{T}}P^{\mathrm{T}}(PP^{\mathrm{T}})^{-1}P\tilde{\theta} \leqslant \frac{1}{\chi}e_f^{\mathrm{T}}e_f \\ &= \frac{1}{\chi}e_f^2, \end{aligned} \tag{47}$$

where

$$0 < \chi \leqslant \frac{2\lambda_{\min}(PP^{\mathrm{T}})}{\lambda_{\max}(\Gamma_1^{-1})}. \tag{48}$$

Thus, (45) can be rewritten as

$$\begin{aligned} &-\tilde{\theta}^{\mathrm{T}}\Gamma_2 e_f - \tilde{\theta}^{\mathrm{T}}\Gamma_3\,\mathrm{sig}(e_f)^r \\ &\leqslant -\frac{\lambda_{\min}(\Gamma_2)}{\lambda_{\max}(P^{\mathrm{T}})}e_f^{\mathrm{T}}e_f - \frac{\lambda_{\min}(\Gamma_3)}{\lambda_{\max}(P^{\mathrm{T}})}(e_f^{\mathrm{T}}e_f)^{(1+r)/2} \\ &\leqslant -\vartheta_1 V_2 - \vartheta_2 V_2^{(1+r)/2}, \end{aligned} \tag{49}$$

where $\vartheta_1 = \frac{\lambda_{\min}(\Gamma_2)\chi}{\lambda_{\max}(P^{\mathrm{T}})}$, $\vartheta_2 = \frac{\lambda_{\min}(\Gamma_3)\chi^{(1+r)/2}}{\lambda_{\max}(P^{\mathrm{T}})}$.

For $0<r<1$, $0<(1+r)/2<1$, from (42), (43) and (49) while noting Lemma B1, we have

$$\begin{aligned}\dot{V} &\leqslant -\rho_1(V_1+V_2)-\rho_2(V_1^{(1+r)/2}+V_2^{(1+r)/2})\\ &\leqslant -\rho_1 V-\rho_2 V^{(1+r)/2},\end{aligned} \tag{50}$$

where $\rho_1=\min\{k_1',\vartheta_1\}$, $\rho_2=\min\{k_2',\vartheta_2\}$.

Therefore, according to Lemma B5, the system states can reach the sliding mode σ_2 within finite time when $\sigma_2\neq 0$ and $\dot{\sigma}_1+c\sigma_1\neq 0$.

Case 2: $\sigma_2\neq 0$ and $\dot{\sigma}_1+c\sigma_1=0$.

For $\sigma_2\neq 0$ and $\dot{\sigma}_1+c\sigma_1=0$, we have $\sigma_2=\sigma_1$, $\dot{\sigma}_2=\dot{\sigma}_1=-c\sigma_1$ and $\varphi(\sigma_1)=0$ from (5), (7) and (12). Then the derivative of V_1 becomes

$$\dot{V}_1=\sigma_2\theta_1\dot{\sigma}_1=\sigma_1\theta_1\dot{\sigma}_1=-c\theta_1\sigma_1^2=-2cV_1<0. \tag{51}$$

If the system states retain in $\sigma_2\neq 0$ and $\dot{\sigma}_1+c\sigma_1=0$, only exponential stability can be obtained from Equation (51). We will prove that the sliding mode σ_2 is reached in finite time in this case.

Substituting control law (9)–(13) into system (3) with some manipulations yields

$$\theta_1\dot{e}_2=\tilde{\theta}^{\mathrm{T}}\varphi_1-\theta_1 c\dot{\sigma}_1-k_1\sigma_2-k_2\sigma_2^r-k_3\,\mathrm{sign}(\sigma_2)-\Delta', \tag{52}$$

where $e_2=x_2-\dot{x}_d$. Denoting $d(\sigma_1)=\dot{\sigma}_1+c\sigma_1$, then (52) can be rewritten as

$$\theta_1\dot{d}(\sigma_1)=\tilde{\theta}^{\mathrm{T}}\varphi_1-k_1\sigma_2-k_2\sigma_2^r-k_3\,\mathrm{sign}(\sigma_2)-\Delta'. \tag{53}$$

For $\sigma_2>0$, $-k_3\,\mathrm{sign}(\sigma_2)-\Delta'=-k_3-\Delta'<0$ as $k_3>\varsigma$, we get

$$\theta_1\dot{d}(\sigma_1)\leqslant\tilde{\theta}^{\mathrm{T}}\varphi_1-k_1\sigma_2-k_2\sigma_2^r. \tag{54}$$

Noting that $\varphi_2=0$ when $\dot{\sigma}_1+c\sigma_1=0$, then the derivative of V_2 satisfies

$$\begin{aligned}\dot{V}_2 &=\tilde{\theta}^{\mathrm{T}}\Gamma_1^{-1}\mathrm{proj}_{\hat{\theta}}\Big(-\Gamma_1\Gamma_2 e_f-\Gamma_1\Gamma_3\,\mathrm{sig}(e_f)^r\Big)\\ &\leqslant -\tilde{\theta}^{\mathrm{T}}\Gamma_2 e_f-\tilde{\theta}^{\mathrm{T}}\Gamma_3\,\mathrm{sig}(e_f)^r\\ &\leqslant -\vartheta_1 V_2-\vartheta_2 V_2^{(1+r)/2}.\end{aligned} \tag{55}$$

According to Lemma B5, we know that $\tilde{\theta}$ converges to zero in finite time. Substituting $\tilde{\theta}=0$ into (54), then

$$\theta_1\dot{d}(\sigma_1)\leqslant -k_1\sigma_2-k_2\sigma_2^r<0. \tag{56}$$

While $\theta_1>0$, $\dot{d}(\sigma_1)<0$. For $\sigma_2<0$, it can be verified that $\dot{d}(\sigma_1)>0$ with the similar process. As shown in Feng et al. (2002), $\sigma_2\neq 0$ and $\dot{\sigma}_1+c\sigma_1=0$ is not an attractor of the system. The system states cannot retain in $\sigma_2\neq 0$ and $\dot{\sigma}_1+c\sigma_1=0$ forever. Thus, the sliding mode σ_2 can be reached in finite time.

Case 3: $\sigma_2=0$.

When $\sigma_2=0$, the system states reach the sliding mode σ_2. If $\sigma_2=0$ at t_0, then $e_1=0$, $\forall t\geqslant t_1$, with t_1 given by (Yu et al. 2008)

$$t_1=\frac{2-\gamma}{c(1-\gamma)}\Big[\ln\big(c|e_1(t_0)|^{\frac{1-\gamma}{2-\gamma}}+\beta''\big)-\ln\beta''\Big]+t_0, \tag{57}$$

where

$$\beta''=\left(\frac{2-\gamma}{\beta}\right)^{\frac{1}{2-\gamma}},$$

and $e_1(t_0)$ is the trajectory error at t_0.

From the analysis in case 1 and case 2, it is verified that the system states can reach the sliding mode σ_2 in finite time. Once the system states reach the sliding mode σ_2, the system states will converge to the equilibrium point along the sliding mode in fast finite-time form as shown in case 3. This completes the proof of (ii). □

Remark 4.1: Noting (27), the adaptive law in (17) contains the estimated-error information of the parameters, which has the same peculiarities as the composite adaptive law. However, it is different from that in Barambones and Etxebarria (2001, 2002), where the prediction error e_f is defined as $e_f=\varphi_f^{\mathrm{T}}\hat{\theta}-u_f$, and the PE condition is required to ensure the finite-time convergence of the trajectory error. In (17), only the positive definiteness of the matrix P after some time is needed to achieve the finite-time control. Due to the integration of the filtered system states, the positive definiteness of P can be satisfied more easily than the PE condition.

Remark 4.2: In Barambones and Etxebarria (2001, 2002), only finite-time control in form of (B5) is realised. By analogy with the difference between TSM and FTSM, the 'fast' finite-time control in form of (B7) possess faster convergence than the finite-time control in form of (B5) when the system states are far away from the equilibrium point. In this article owning to

Table 1. Parameters of EMA.

Parameters	J (kg m^2)	M_j^δ (N m rad^{-1})	f (N m (rad s^{-1})$^{-1}$))	M_c (N m)	K_1 (N m V^{-1})
Quantities	7.56	305	9.12	4.12	28.23

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the utilisation of NFTSMC and composite adaptive law in (17), the 'fast' finite-time control in form of (B7) is obtained.

Remark 4.3: In view of the positive definiteness of P, the parameter ε_1 in (23) must be carefully designed. A large ε_1 is not recommended as it renders a rapid attenuation of $\ell(\tau)$, with which some useful information may be lost in P and makes it difficult to be positive definite. Generally a small positive number is selected.

Remark 4.4: In (17) if $\Gamma_3 = 0$, then the adaptive law which is similar to that in Zhang et al. (2009) can be obtained. But when $\Gamma_3 = 0$, $\vartheta_2 = 0$ in (49) and (55), and $\rho_2 = 0$. So $\dot{V} \leqslant -\rho_1 V$, and only exponential stability can be gained. Here with the employment of the nonlinear term $\mathrm{sig}(e_f)^r$ in (17) and the NFTSMC approach, the finite-time control is obtained.

5. Simulation and experiment

In this section, the effectiveness of the proposed method is validated by simulation and experimental study. The identified parameters of the EMA system (which is introduced in Section 5.2) given in Table 1 will be used in both simulation and experimental study.

5.1. *Simulation and analysis*

The unmodelled dynamics is set as $\Delta = [2\,\mathrm{rand}(1) - 1]$ N m in simulation, where rand(1) is random number in [0, 1]. According to (4) with the values presented in Table 1, the parameter vector θ are given by: $\theta = [0.268, 10.806, 0.319, 0.146]^{\mathrm{T}}$. The parameter bounds $\theta_{\min}$ and $\theta_{\max}$ are as follows: $\theta_{\min} = [0.05, -15, 0.02, 0]^{\mathrm{T}}$, $\theta_{\max} = [0.5, 15, 0.6, 0.6]^{\mathrm{T}}$.

Four control methods, i.e., the proposed ANFTSMC and ANFTSMC with adaptive law in Barambones and Etxebarria (2001, 2002), the NTSMC (Feng et al. 2002) and NFTSMC (Yu et al. 2008) are implemented. To avoid the chattering phenomena saturation function with the form of $(2/\pi)\mathrm{atan}(900\sigma_2)$ instead of $\mathrm{sign}(\sigma_2)$ is used in $u_{s,2}$ for ANFTSMC while in u for NFTSMC and NTSMC. As in practical systems, the control effort is limited by the voltage supply source. Here the control effort is limited in the interval of $-10 \sim +10$ V.

The following values have been chosen for the parameters of the proposed ANFTSMC: $\beta = 0.1$, $c = 10$, $k_1 = 150$, $k_2 = 150$, $k_3 = 1.5$, $\gamma = r = 13/15$, $\kappa_f = 100$, $\varepsilon_1 = 0.25$, $\Gamma_1 = \mathrm{diag}(1, 60, 3, 2.5)$, $\Gamma_2 = \Gamma_3 = \mathrm{diag}(5, 50, 2.5, 1.5)$. The initial parameter estimates are set as follows. (C1): $\hat{\theta}(0) = [0.12, 0, 0.15, 0]^{\mathrm{T}}$. (C2): $\hat{\theta}(0) = \theta_{\max}$.(C3): $\hat{\theta}(0) = \theta_{\min}$.

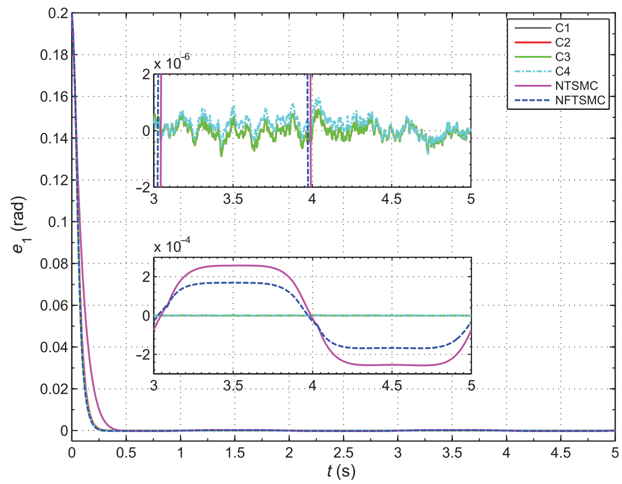

Figure 1. Tracking efforts with sinusoidal excitation.

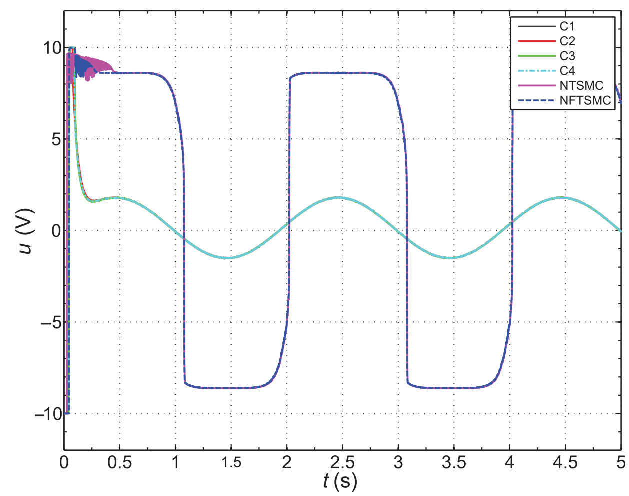

Figure 2. Control efforts with sinusoidal excitation.

Noting Remark 4.1, if we let $e_f = \varphi_f^{\mathrm{T}}\hat{\theta} - u_f$, and define the adaptive law as $\dot{\hat{\theta}} = \mathrm{proj}_{\hat{\theta}}\big(\Gamma_1\varphi_2\sigma_2 - \Gamma_1\Gamma_2\varphi_f e_f - \Gamma_1\Gamma_3\varphi_f\,\mathrm{sig}(e_f)^r\big)$, then adaptive law in Barambones and Etxebarria (2001, 2002) will be used. For simplicity, we denote ANFTSMC with adaptive law in Barambones and Etxebarria (2001, 2002) as 'C4', and the initial parameter estimates is $\hat{\theta}(0) = [0.12, 0, 0.15, 0]^{\mathrm{T}}$. Controller parameters are the same as those of the proposed ANFTSMC except for $\Gamma_1 = \mathrm{diag}(0.15, 5, 0.2, 0.1)$, $\Gamma_2 = \Gamma_3 = \mathrm{diag}(0.2, 8, 6, 1.2)$, which are tuned to gain the best parameter estimation.

The values for the parameters of NFTSMC are as follows: $\beta = 0.1$, $c = 10$, $\gamma = 13/15$, $k = 10$. If we let $c = 0$ in (5), then the nonsingular terminal sliding mode in Feng et al. (2002) and Yu et al. (2005) can be gained.

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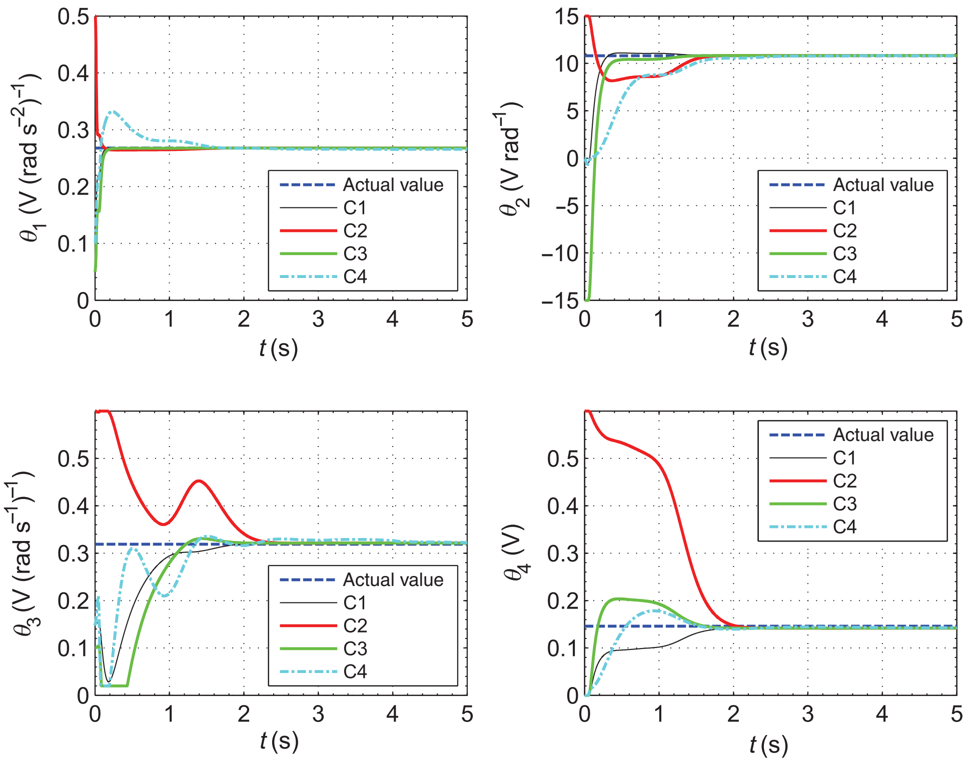

Figure 3. Estimates of parameters with sinusoidal excitation.

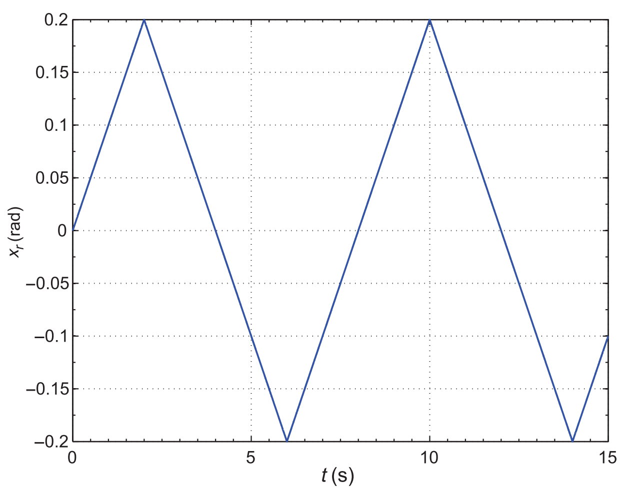

Figure 4. Reference signal for triangular excitation.

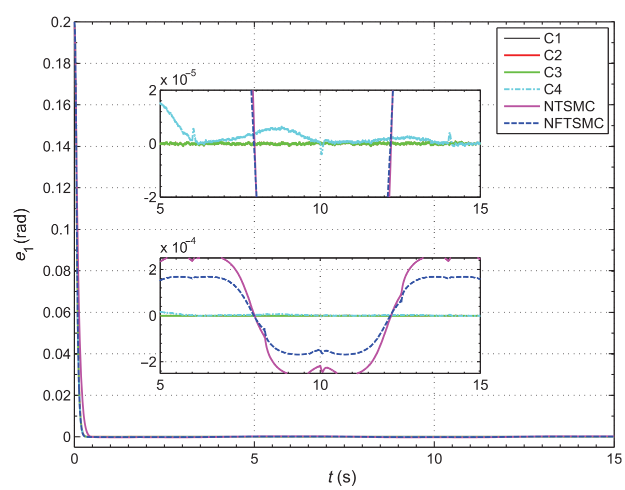

Figure 5. Tracking efforts with triangular excitation.

So the control law of NTSMC can be gained by setting $c = 0$ in NFTSMC.

The simulation contains two sets. In one set the desired trajectory is $x_d = 0.2\sin(\pi t)$rad, and the initial angular is set as 0.2 rad. Figure 1 shows the trajectory errors of the output angle. For ANFTSMCs, the trajectory error attenuates as quickly as that of NFTSMC, while in NTSMC it attenuates slower than that in NFTSMC and ANFTSMCs. Nevertheless, both NFTSMC and NTSMC hold larger control effort than that of ANFTSMCs as shown in Figure 2, which verifies the conservativeness of NFTSMC and NTSMC. In Figure 1, we can see the trajectory error converge to a region around the equilibrium in all control methods as the saturation function instead of the signum function is used in the control law (Feng, Han, Yu, Stonier, and Man 2000). However, the steady-state error of ANFTSMCs is

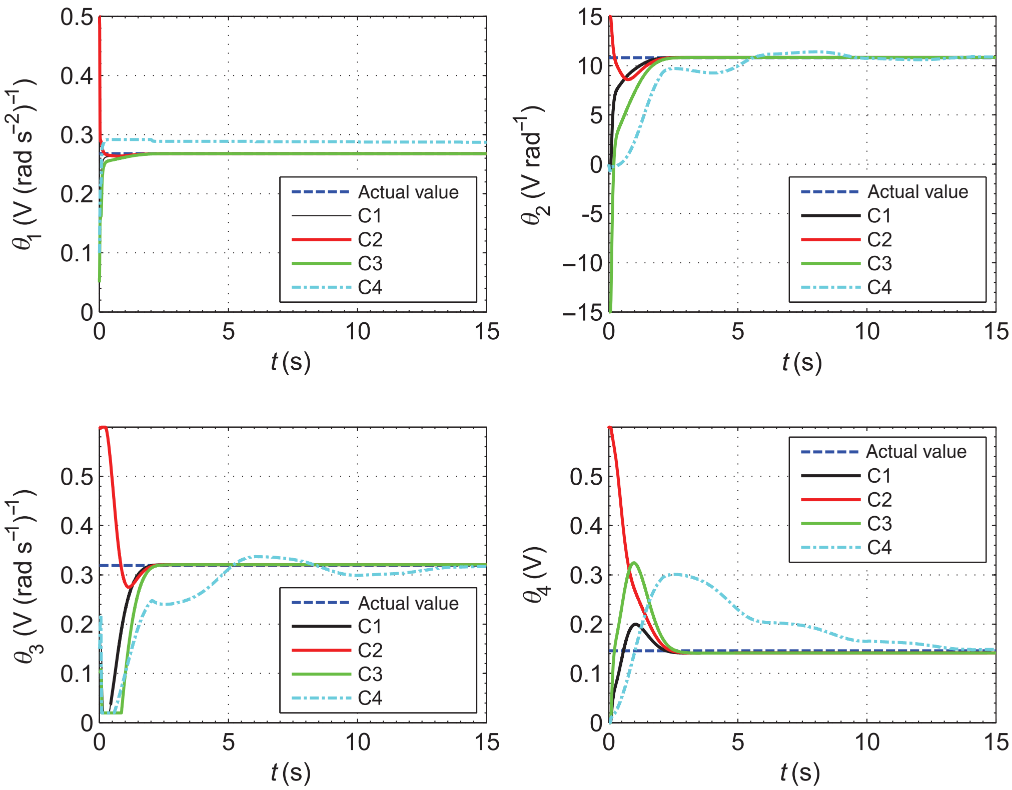

Figure 6. Estimation of parameters with triangular excitation.

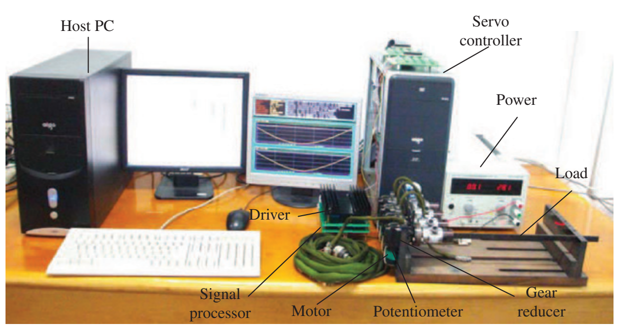

Figure 7. EMA experimental study facility.

smaller than that of NFTSMC and NTSMC. In ANFTSMCs only the robust control term $u_{s,2}$ is affected by saturation function, while in NFTSMC and NTSMC, the whole control effort is influenced, thus the NFTSMC and NTSMC methods possess larger steady-state error than ANFTSMCs with the same saturation function. Figure 3 shows the estimates of the parameters in ANFTSMCs. For the proposed ANFTSMC, the estimates tend to the real values fast despite the different initial values as is observed. While for ANFTSMC with adaptive law in Barambones and Etxebarria (2001, 2002), the estimates also tend to the real values, but not as quickly and smoothly as those in the proposed ANFTSMC.

In another set the desired trajectory is obtained by putting the reference signal shown in Figure 4 to pass through a second-order filter. The reference signal (which is named as x_r) is symmetrical triangular signal whose amplitude is 0.2 rad and the period is 8 s. The second-order filter with the transfer function model $\frac{2500}{s^2+100s+2500}$ is used to gain the bounded desired trajectory (Assumption 3), where s is the Laplace operator. The initial angular of the plant is 0.2 rad. Figure 5 shows that the proposed ANFTSMC can gain the best trajectory tracking in all the control methods. In Figure 6, it shows that the estimates of the parameters in the proposed ANFTSMC also tend to the real values with different initial values under the triangular excitation. But in the adaptive law in Barambones and Etxebarria (2001, 2002), it is more difficult for the estimates of the parameters tend to the real values than those under the sinusoidal excitation.

From Figures 3 and 6, we can see that the proposed adaptive law can achieve better parameter estimation than that in Barambones and Etxebarria (2001, 2002). The reason is that in comparison with the PE condition required in Barambones and Etxebarria (2001, 2002), the positive definiteness of P

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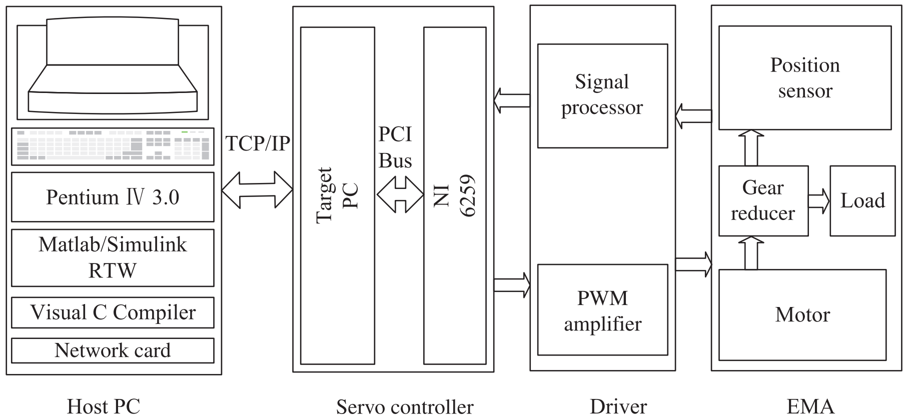

Figure 8. Block diagram of the experimental architecture.

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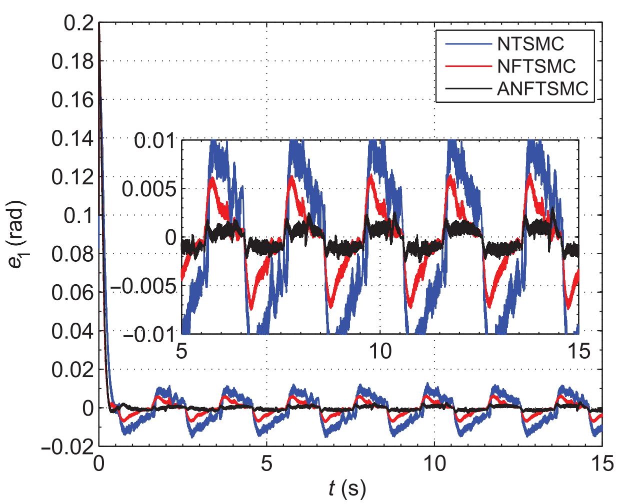

Figure 9. Tracking error of the system in experiment.

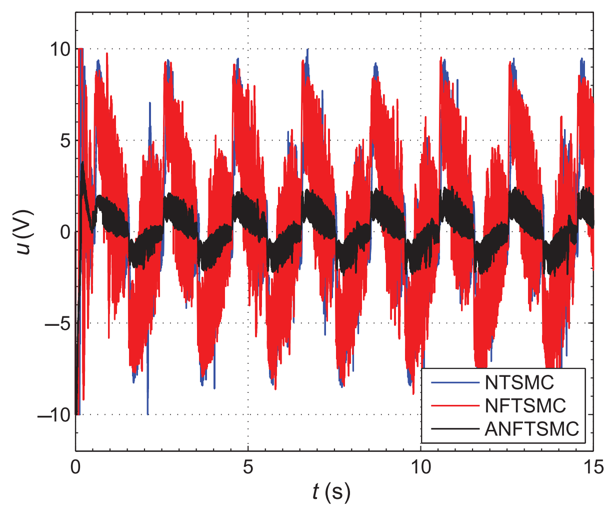

Figure 10. Control efforts in experiment.

in the proposed ANFTSMC can be more easily satisfied (Remark 4.1).

5.2. *Experimental study*

To demonstrate the effectiveness of the proposed ANFTSMC, an EMA system is set up as a testing-bed. As shown in Figure 7, the testing-bed comprises five major components: an EMA, a driver, a signal processor, a servo controller and a host PC. The EMA consists of a DC motor, a gear reducer, a potentiometer and the load. In the EMA the motor drives the load via the gear reducer. The driver contains a PWM amplifier that can drive the motor, and the signal processor can acquire the angular information of the EMA with the potentiometer that is fixed on the gear reducer.

The controller of the servo system is implemented through an Xpc target that consists of a target personal computer and the interface card NI PCI-6259 (Figure 8). The sampling time of the servo controller is 1 ms, a value in common use for servo mechanisms. The input and output range of the card are set as $-10\sim+10$V.

Identification is performed to obtain the parameters, whose values are shown in Table 1. Three control methods, i.e. the proposed ANFTSMC, the NTSMC and the NFTSMC are implemented. Parameters of the controllers are the same as those in simulation. For ANFTSMC, the initial parameter values is set as $\hat{\theta}(0) = [0.12, 0, 0.15, 0]^{\mathrm{T}}$. The desired trajectory is selected as $x_d = 0.2\sin(\pi t)$ rad, and the initial angular is 0.2 rad.

Tracking errors are shown in Figure 9. It shows that the tracking error of ANFTSMC is smaller than

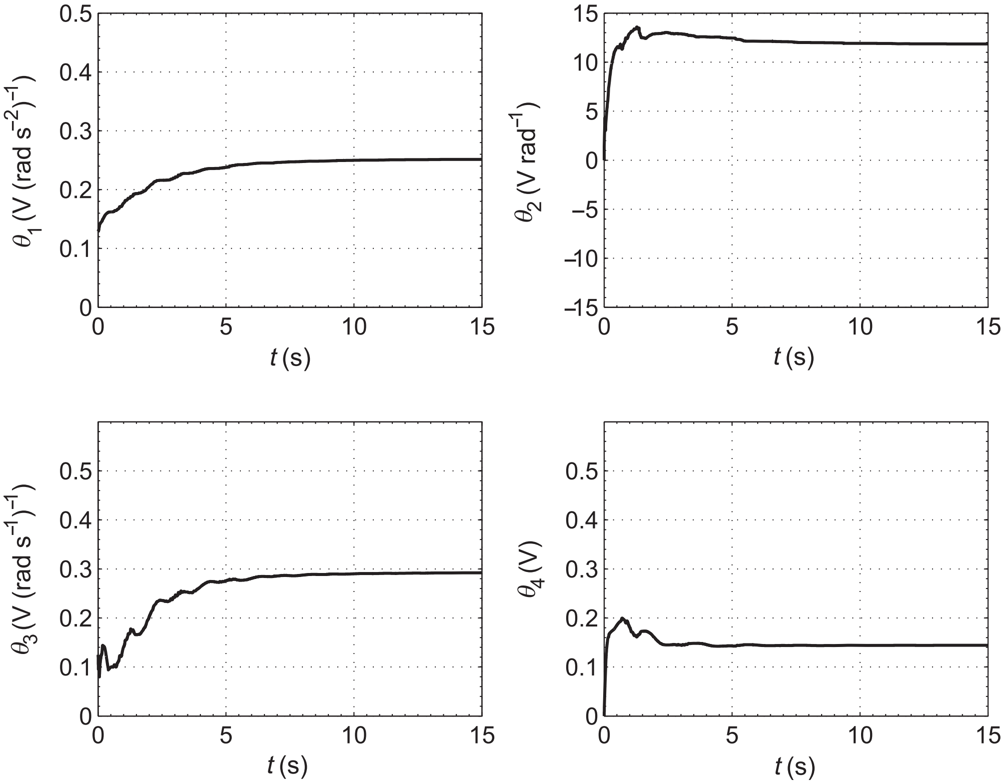

Figure 11. Estimation of parameters in experiment.

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that of NTSMC and NFTSMC, and the tracking error of ANFTSMC and NFTSMC attenuates faster than that of NTSMC. Figure 10 shows that ANFTSMC holds a small control effort. Thus, the experiment results coincide with those of the simulation. Parameter estimates are shown in Figure 11. As for noise exists in the testing-bed, the estimates of the parameters may not tend to the real values as precisely as those in simulation. But it can be seen that the parameter estimates tend to steady values quickly.

6. Conclusion

In this article, an ANFTSMC scheme for EMA has been presented. The control scheme consists of an adaptive control term with improved composite adaptive law and a robust control term with a modified NFTSMC approach. The adaptive control term adopts a composite adaptive law where the integration of filtered system states is used to estimate the uncertain parameters, and fast parameter convergence can be achieved. The estimates are then used as controller parameters to overcome the effects of modelled uncertainties. While the robust control term which is based on NFTSMC with a fast TSM reaching law provides fast finite-time convergence of errors either far away from or near the equilibrium point. Duo to the parameter adaption, a small switching gain is used to robustify the design against unmodelled uncertainties. Conclusively, the control scheme assures the robustness in both the parameter uncertainties and external disturbances without a high gain control effort that has to be utilised in NFTSMC. Furthermore, the control method enables the elimination of singular problem in conventional TSMC and FTSMC. It has been proved that the closed-loop system with the proposed ANFTSMC is stable, and the tracking errors converges to zero in fast finite-time form if the nonsingularity of the matrix P holds. Finally, it has been shown by simulation and experimental study that the proposed control scheme performs reasonably well and the tracking control objective is achieved.

Acknowledgements

This work was supported by the National Natural Science Fund of China (Grant: 60972118, 3060021500904), the National Science Fund for Distinguished Young Scholars of China (Grant: 60925011), Fund of Ordinary University Key Lab. of Beijing (Automatic Control System) (Grant: 3060012250903). The authors thank the reviewers for their very helpful comments and suggestions which have improved the presentation of this article.

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References

Barambones, O., and Etxebarria, V. (2001), 'Robust Sliding Composite Adaptive Control for Mechanical Manipulators with Finite Error Convergence Time', *International Journal of Systems Science*, 32, 1101–1108.

Barambones, O., and Etxebarria, V. (2002), 'Energy-based Approach to Sliding Composite Adaptive Control for Rigid Robots with Finite Error Convergence Time', *International Journal of Control*, 75, 352–359.

Bhat, S.P., and Bernstein, D.S. (1998), 'Continuous Finite Time Stabilisation of the Translational and Rotational Double Integrators', *IEEE Transactions on Automatic Control*, 43, 678–682.

Bhat, S.P., and Bernstein, D.S. (2000), 'Finite-time Stability of Continuous Autonomous Systems', *SIAM Journal on Control and Optimization*, 38, 751–766.

Feng, Y., Han, F., Yu, X., Stonier, D., and Man, Z. (2000), 'Tracking Precision Analysis of Terminal Sliding Mode Control Systems with Saturation Functions', in *Advances in Variable Structure Systems: Analysis, Integration and Applications*, eds. X. Yu and J.X. Xu, Singapore: World Scientific, pp. 325–334.

Feng, Y., Yu, X., and Man, Z. (2002), 'Non-singular Terminal Sliding Mode Control of Rigid Manipulators', *Automatica*, 38, 2159–2167.

Hale, J.K. (1969), *Ordinary Differential Equations*, Huntington, NJ: Krieger.

Ilyas, E. (2006), 'Sliding Mode Control with PID Sliding Surface and Experimental Application to an Electromechanical Plant', *ISA Transactions*, 45, 109–118.

Ioannou, P.A., and Sun, J. (1996), *Robust Adaptive Control,* Upper Saddle River, NJ: Prentice-Hall.

Jin, M., Lee, J., Chang, P.H., and Choi, C. (2009), 'Practical Nonsingular Terminal Sliding-mode Control of Robot Manipulators for High-accuracy Tracking Control', *IEEE Transactions on Industrial Electronics*, 56, 3593–3691.

Keleher, P.G., and Stonier, R.J. (2002), 'Adaptive Terminal Sliding Mode Control of a Rigid Robotic Manipulator with Uncertain Dynamics Incorporating Constraint Inequalities', *ANZIAM Journal*, 43, E102–E152.

Man, Z., Mike, O.D., and Yu, X. (1999), 'A Robust Adaptive Terminal Sliding Mode Control for Rigid Robotic Manipulators', *Journal of Intelligent and Robotic Systems*, 24, 23–41.

Man, Z., and Yu, X. (1997), 'Terminal Sliding Mode Control of MIMO Linear Systems', *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 44, 1065–1070.

Mitrinovic, D.S. (1970), *Analytic Inequalities*, New York: Springer.

Slotine, J.J.E., and Li, W. (1989), 'Composite Adaptive Control of Robot Manipulators', *Automatica*, 25, 509–519.

Song, G., Longman, R.W., and Mukherjee, R. (1999), 'Integrated Sliding-mode Adaptive-robust Control', *IEE Proceedings–Control Theory and Application*, 146, 341–347.

Venkataraman, S.T., and Gulati, S. (1992), 'Control of Nonlinear Systems Using Terminal Sliding Modes', in *Proceedings of American Control Conference*, pp. 891–893.

Utkin, V.I. (1977), 'Variable Structure Systems with Sliding Modes', *IEEE Transactions on Automatic Control*, AC-22, 212–222.

Utkin, V.I. (1993), 'Sliding Mode Control Design Principles and Applications to Electric Drives', *IEEE Transactions on Industrial Electronics*, 40, 23–36.

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Utkin, V.I., Guldncr, J., and Shi, J. (1999), *Sliding Mode Control in Electromechanical Systems*, London: Taylor & Francis.

Wu, S., and Fei, Y. (2005), *Flying Control System*, Beijing: Press of Beijing University of Aeronautics and Astronautics.

Wu, Y., Yu, X., and Man, Z. (1998), 'Terminal Sliding Mode Control Design for Uncertain Dynamic Systems', *Systems and Control Letters*, 34, 281–288.

Young, K.D., Utkin, V.I., and Ozguner, U. (1999), 'A Control Engineer's Guide to Sliding Mode Control', *IEEE Transactions on Control Systems Technology*, 7, 328–342.

Yu, S., Du, J., Yu, X., and Xu H. (2008), 'A Novel Recursive Terminal Sliding Mode with Finite-tIme Convergence', in *Proceedings of the 17th World Congress, the International Federation of Automatic Control*, pp. 5945–5949.

Yu, X., and Man, Z. (1996), 'Model Reference Adaptive Control Systems with Terminal Sliding Modes', *International Journal of Control*, 64, 1165–1176.

Yu, X., and Man, Z. (2002), 'Fast Terminal Sliding-mode Control Design for Nonlinear Dynamical Systems', *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 49, 261–264.

Yu, S., Yu, X., Shirinzadeh, B., and Man, Z. (2005), 'Continuous Finite-time Control for Robotic Manipulators with Terminal Sliding Mode', *Automatica*, 41, 1957–1964.

Zak, M. (1989), 'Terminal Attractors in Neural Networks', *Neural Networks*, 2, 259–274.

Zhao, D., Li, S., and Gao, F. (2009), 'Finite Time Position Synchronised Control for Parallel Manipulators using Fast Terminal Sliding Mode', *International Journal of Systems Science*, 40, 829–843.

Zhao, D., Li, S., Gao, F., and Zhu, Q. (2009), 'Robust Adaptive Terminal Sliding Mode-based Synchronised Position Control for Multiple Motion Axes Systems', *IET Control Theory Application*, 3, 136–150.

Zhang, G., Chen, J., and Li, Z. (2009), 'An Adaptive Robust Control for Linear Motors Based on Composite Adaptation', *Control Theory and Application*, 26, 833–837.

Zhang, G., Chen, J., and Li, Z. (2010), 'Identifier Based Adaptive Robust Control for Servo Mechanisms with Improved Transient Performance', *IEEE Transactions on Industrial Electronics*, 57, 2536–2547.

Appendix A: TSM in Barambones and Etxebarria (2001, 2002) and Zhao et al. (2009)

In Zhao et al. (2009) the sliding mode is defined as $\sigma = \dot{x} - x_r$, and x_r is as

$$\begin{cases} x_r = \dot{x}_d - \Lambda_1 e - \Lambda_2 \operatorname{sig}(e)^\gamma, \\ \dot{x}_r = \ddot{x}_d - \Lambda_1 e - \Lambda_2 e_r, \end{cases} \tag{A1}$$

where $e_r = [e_{r1}, \dots, e_{rn}]^{\mathrm{T}}$, and $e_{ri} (i = 1, 2, \dots, n)$ is defined as

$$e_{ri} = \begin{cases} |e_i|^{\gamma-1} \dot{e}_i, & e_i \neq 0, \dot{e}_i \neq 0, \\ |\varepsilon|^{\gamma-1} \dot{e}_i, & e_i = 0, \dot{e}_i \neq 0, \\ 0, & e_i = 0. \end{cases} \tag{A2}$$

It can be proved that with the definition in (A1) and (A2) the fast TSM is achieved. If $\Lambda_1 = 0$ and $\varepsilon = 0$, TSM in Barambones and Etxebarria (2001, 2002) can be gained. However, when $e_i \to 0$ and $\dot{e}_i \neq 0$, e_{ri} may become awfully large. The global nonsingularity cannot be achieved.

Appendix B: Preliminaries

Some definitions, lemmas used are introduced in this section.

Lemma B1: *Assume $a_1 > 0$, $a_2 > 0$ and $0 < b < 1$, then the following inequality holds* (Mitrinovic 1970):

$$(a_1 + a_2)^b \leqslant a_1^b + a_2^b. \tag{B1}$$

Lemma B2: *Suppose $a_1 > 0$, $a_2 > 0, \dots, a_n > 0$ and $0 < q < 2$, then the following inequality holds* (Yu et al. 2005):

$$(a_1^2 + a_2^2 + \cdots + a_n^2)^q \leqslant (a_1^q + a_2^q + \cdots + a_n^q)^2. \tag{B2}$$

Definition B1: If $\Phi(V(t), t)$ is a scalar function of scalars $V(t)$, t in some open connected set D, then a function $V(t)$, $t_0 \leqslant t < t_1$, $t_1 > t_0$ is a solution of the differential inequality

$$\dot{V}(t) \leqslant \Phi(V(t), t), \tag{B3}$$

on $[t_0, t_1)$ if $V(t)$ is continuous on $[t_0, t_1)$ and its derivative on $[t_0, t_1)$ satisfies (B3) (Hale 1969).

Lemma B3: *Let $\Phi(d(t), t)$ be continuous on an open connected set $D \in R^2$ and assume that the initial value problem for the scalar equation*

$$\dot{d}(t) = \Phi(d(t), t), \quad d(t_0) = d_0, \tag{B4}$$

has a unique solution. If d(t) *is a solution of* (B4) *on $t_0 \leqslant t < t_1$ and $V(t)$ is a solution of* (B3) *on $t_0 \leqslant t < t_1$ with $V(t_0) \leqslant d(t_0)$, then $V(t) \leqslant d(t)$ for $t_0 \leqslant t < t_1$* (Hale 1969).

Lemma B4: *Assume that a continuous positive-definite function $V(t)$ satisfies the following differential inequality* (Barambones and Etxebarria 2001, 2002)

$$\dot{V}(t) \leqslant -\alpha V^\mu \quad \forall t \geqslant t_0, \quad V(t_0) \geqslant 0, \tag{B5}$$

where $\alpha > 0$, $0 < \mu < 1$ are constants. Then $V(t) = 0$ $\forall t \geq t_1$ with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\mu}(t_0)}{\alpha(1-\mu)}. \tag{B6}$$

Lemma B5: *Assume that a continuous positive-definite function $V(t)$ satisfies the following differential inequality*:

$$\dot{V}(t) \leqslant -\alpha_1 V - \alpha_2 V^\mu \quad \forall t \geqslant t_0, \quad V(t_0) \geqslant 0, \tag{B7}$$

where $\alpha_1 > 0$, $\alpha_2 > 0$, $0 < \mu < 1$ are constants. Then $V(t) = 0$, $\forall t \geqslant t_1$ with t_1 given by

$$t_1 = t_0 + \frac{1}{\alpha_1(1-\mu)} \{\ln[\alpha_1 V^{1-\mu}(t_0) + \alpha_2] - \ln \alpha_2\}. \tag{B8}$$

Proof: Consider the following differential equation:

$$\dot{d}(t) = -\alpha_1 d - \alpha_2 d^{\mu}, \quad d(t_0) = V(t_0). \tag{B9}$$

The unique solution to this equation can be found as

$$\ln[\alpha_1 d^{1-\mu}(t) + \alpha_2] = \ln[\alpha_1 d^{1-\mu}(t_0) + \alpha_2] - \alpha_1(1-\mu)(t - t_0). \tag{B10}$$

Therefore from Lemma B1, we have

$$\begin{aligned} &\ln[\alpha_1 V^{1-\mu}(t) + \alpha_2] \\ &\quad \leqslant \ln[\alpha_1 d^{1-\mu}(t) + \alpha_2] \\ &\quad = \ln[\alpha_1 d^{1-\mu}(t_0) + \alpha_2] - \alpha_1(1-\mu)(t - t_0), \quad t_0 \leqslant t < t_1, \end{aligned} \tag{B11}$$

and $V(t) = 0\ \forall t \geqslant t_1$, with t_1 given in (B8). This completes the proof of Lemma B5.

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