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Emerald Article: Distributed rigid formation control algorithm for multi-agent systems

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Distributed rigid formation control algorithm for multi-agent systems

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Abstract

Purpose – Rigidity of formation is an important concept in multi-agent localization and control problems. The purpose of this paper is to design the control laws to enable the group to asymptotically exhibit the flocking motion while preserving the network rigidity at all times.

Design/methodology/approach – The novel approach for designing control laws is derived from a smooth artificial potential function based on an undirected infinitesimally rigid formation which specifies the target formation. Then the potential function is used to specify a gradient control law, under which the original system then becomes an orderly infinitesimally rigid formation.

Findings – The strong relationship between the stability of the target formation and the gradient control protocol are utilized to design the control laws which can be proved to make the target formation stable. However, the rigidity matrix is not utilized in the design of control law. Future research will mainly focus on formation control with the relationship of rigidity matrix.

Originality/value – The value of this paper is focused on the control laws design and the control laws could enable the group to asymptotically exhibit the flocking motion while preserving the network rigidity at all times. Also the detailed simulations and experiments are given to prove that the novel approach is available.

Keywords Control systems, Programming and algorithm theory, Robots, Graph rigidity, Multi-agent systems, Gradient control, Formation control, Potential function, Flocking control

Paper type Research paper

1. Introduction

Formation control of multi-robot networks is an area of ongoing research in control systems as witnessed by an increasing number of contributions in recent years. Among the older contributions, we note, Cortes (2008), Lee and Spong (2006), Tanner *et al.* (2003a, b), Wen *et al.* (2010), Francisco and Juan (2006) and Emilie and Yves (2001). Formation problems are particularly interesting due to their possible application in multi-robot networks formed using reconfigurable sensor networks. A creative extension of sensor networks is to make the network devices mobile, creating a reconfigurable sensor network which immediately gives rise to a multi-agent control problem (Olfati-Saber, 2006; Hur and Ahn, 2010; Fidan *et al.*, 2007; Anderson *et al.*, 2008). Maintaining a specific formation for a reconfigurable sensor network is more and more necessary in order to gather more data.

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In the formation control problem, graph theory plays a natural role and especially the study of graph rigidity has intrigued lots of mathematicians and researchers (Laman, 1970; Jackson and Jordan, 2005; Fang and Morse, 2008; Dimarogonas and Johansson, 2008, 2009; Zhang and Leonard, 2006). The potential function approaches are generally used to design distributed control laws in rigidity formation control. Aspnes *et al.* (2006) studies a problem that is independent of the reconfigurability of the network. The network localization problem is solvable if three beacons are in a general position and the graph of known distances is generically globally rigid. The use of rigidity in Aspnes *et al.* (2006) shows the interesting relationship between the concept of rigidity and problems involving formations. This leads naturally to the idea of designing a formation control based on rigid graphs. This is an approach to formation control taken by Olfati-Saber and Murray (2002). They use a double integrator model for point mass robots and they propose using rigid graph theory to define the formation. Olfati-Saber and Murray (2002) also propose a gradient control law involving prescribed distances and they also give a proof of stability based on the LaSalle invariance theorem. The proof does not analyze all equilibrium of the control law. The closed-loop dynamics are not proved to be locally Lipschitz. Also, the control law uses global velocity measurements to stabilize double integrators. Finally, although a set stability result for the equilibrium set is claimed, but there are more simulations and experiments in the equilibrium set. In this paper, we use similar ideas with rigidity theory to formulate a control problem based on distances between agents. As with Olfati-Saber and Murray (2002) and Absil and Kurdyka (2006), a gradient control law is derived and it is shown to be locally asymptotically stable. However, there are important differences between our work and theirs, and these lie in that this paper proposes a novel formation control approach with rigidity theory, and lots of simulations and experiments prove that this novel approach is available.

This paper is organized as follows. In Section 2, we define the notations about graph rigidity and rigid matrix which are used throughout this paper. The gradient control law for rigid formation generation is introduced in Section 3. Then, in Section 4, simulations and experiments are presented to demonstrate its validity as a quantitative measurement of formation rigidity. Finally, conclusions are given in Section 5 with some prospective work related to the work in this paper.

2. Preliminaries

A. Graph rigidity

To introduce graph rigidity we define $G = (V, E)$ to be an undirected complete graph with n vertices. Define the composite vector $p = (p_1, p_2, \ldots p_n) \in R^{2n}$ and a pair (G, p).

Also we define the rigidity function associated with the framework (G, p) as the function $g_G : R^{2n} \rightarrow R^{|E|}$:

$$g_G := (\ldots, \|p_k - p_j\|^2, \ldots) \tag{1}$$

The ith component $\|p_k - p_j\|^2$ corresponds to the edge e_i of E.

There are three similar definitions of rigidity in graph: rigidity, global rigidity, and infinitesimal rigidity (Krick *et al.*, 2008; Asimow and Roth, 1978).

Definition 2.1. A framework (G, p) is *rigid* if there exists a neighborhood $U \subset R^{2n}$ of p such that:

$$g_G^{-1}(g_G(p)) \cap U = g_K^{-1}(g_K(p)) \cap U \tag{2}$$

where K is the complete graph with the same vertices as G.

Definition 2.2. A framework (G, p) is *globally rigid* if $g_G^{-1}(g_G(p)) = g_K^{-1}(g_K(p))$.

Definition 2.3. A framework (G, p) is *infinitesimally rigid* in the plane if $\dim(\ker J_{g_G}(p)) = 3$, or if:

$$\operatorname{rank} J_{g_G}(p) = 2n - 3 \tag{3}$$

where $J_{g_G}(p)$ is *rigidity matrix* and it will be explained in part B.

Definition 2.4. Point p are regular points of the graph G with n vertices if:

$$\operatorname{rank} J_{g_G}(p) = \max\{\operatorname{rank} J_{g_G}(q) | q \in R^{2n}\} \tag{4}$$

From Definition 2.4, we know the following theorem.

Theorem 2.1. (Asimow and Roth, 1978) a framework (G, p) is *infinitesimally rigid if and only if* (G, p) is rigid and p are regular points.

Theorem 2.1 tells us that some framework is rigid but not infinitesimally rigid. However, if the framework is infinitesimally rigid, then it is sure to be rigid. Figure 1 shows these properties with two examples. It is easily to compute that rank $J_{g_G}(q) = 2n - 3$ in Figure 1(a) and rank $J_{g_G}(q) < 2n - 3$ in Figure 1(b), so Figure 1(a) is rigid and infinitesimally rigid; Figure 1(b) is rigid but not infinitesimally rigid, as p are not regular points. In general, the rigid but fail to be infinitesimally rigid graph almost have parallel or collinear edges. In this paper, rigid means infinitesimally rigid.

B. Rigidity matrix

Based on Part A, in framework (G, p) we define $d_{ij} = \|p_i - p_j\|$ the Euclidean distances between pairs of points (p_i, p_j) and they are constant. Also, we can get (Krick *et al.*, 2008):

$$(p_i - p_j) \cdot (p_i - p_j) = d_{ij}^2 \quad i,j \in \{1, 2, \ldots, n\} \tag{5}$$

Assuming a smooth trajectory, equation (4) can be differentiated:

$$2(p_i - p_j) \cdot (\dot{p}_i - \dot{p}_j) = 0, \quad i,j \in \{1, 2, \ldots, n\}, \quad t \geq 0 \tag{6}$$

where $\widehat{p_i}$ is the velocity of point p_i, and we collect equation (5) into a new equation:

$$J_{g_G}(p)\dot{p} = 0 \tag{7}$$

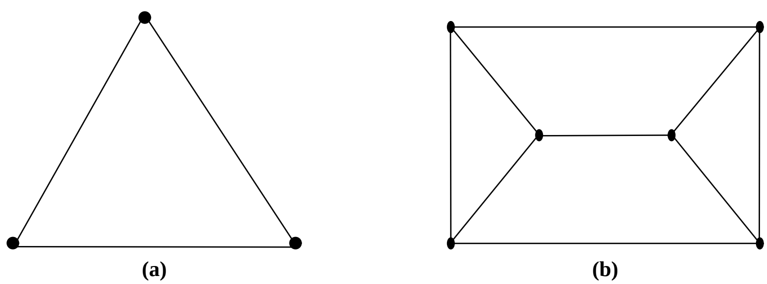


Figure 1.
The two possible examples with rigid framework

where $J_{g_G}(p)\dot{p} = 0$, $\dot{p} = column\ \dot{p}_1, \dot{p}_2, \ldots \dot{p}_n$, and $J_{g_G}(p)$ is the rigidity matrix with structure $m \times nd$ where $m = C_n^2$.

3. Gradient control laws

A. Problem formulation

Consider a group of N agents moving in an n-dimensional Euclidean space, and each one has point mass dynamics. A continuous-time model of the system is described by:

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= u_i \quad i = 1, 2, \ldots, N \end{aligned} \tag{8}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$ is the position vector of agent i, $v_i = (v_{i1}, v_{i2}, \ldots, v_{in})^T \in R^n$ is the velocity vector, $u_i = (u_{i1}, u_{i2}, \ldots, u_{in})^T \in R^n$ is the control input acting on agent i. The relative position vector between agents i and j is represented by $x_{ij} = x_i - x_j$. In order to fulfill the control objective, $u_i \in R^n$ should be designed to enable the group to achieve the desired rigid flocking motion and to preserve the network rigid structure as time evolves.

B. Equations

It is well known that the collective objective of flocking motion can be described by the relative positions and relative velocities of the agents. Hence the desired relative positions are uniquely determined by utilizing the flocking vector $f = (f_1, f_2, \ldots, f_N)^T \in R^{mN}, f_i \in R^m, \forall i$. Define the goal topology of a multi-agent system as $G_g = (V_g, E_g)$, the edge set is given by:

$$E_g = \{(n_i, n_j) \| \|f_i - f_j\| = d_{ij} < R_d, n_i, n_j \in V_g, i \neq j\} \tag{9}$$

where R_d is the communication radius of agents. Therefore, the objective of rigid flocking formation control for multi-agent systems can be directly described as follows.

Definition 3.1. (Stable infinitesimally rigid formation) give a flocking vector $f = (f_1, f_2, \ldots, f_N)^T$ and assume that $G_g = (V_g, E_g)$ is connected. Then a multi-agent system is a stable infinitesimally rigid one iff for any $(n_i, n_j) \in E_g$, it satisfies:

$$\begin{cases} x_i - x_j = f_i - f_j = d_{ij} \\ v_i = v_j \\ \operatorname{rank} J_{g_G}(f) = 2n - 3 \end{cases} \tag{10}$$

On the whole, the main idea of the proposed flocking control strategy is to relate the desired rigid geometric configurations of the goal topology to the local or global extremes of the group potential functions (Schneider and Wildermuth, 2005). The distributed control law u_i is designed by using the state information from agent i and its neighbors hence can be described in the following form:

$$u_i = \underbrace{-\sum_{j \in N_i(G_c)} \nabla_{x_{ij}} V_{ij}(\|x_{ij}\|)}_{u_i^1} \underbrace{-\sum_{j \in N_i(G_c)} a_{ij}(t)(v_i - v_j)}_{u_i^2} \tag{11}$$

where u_i^1 acts as the induced term for achieving the desired rigid configuration, u_i^2 is the velocity consensus term and is responsible for aligning the agent velocities to a common value. V_{ij} represents the artificial potential function between agents i and j. Under the assumption that the communication topology is connected while the robots are in the radius of communication during the whole process of evolution, it can be proved that the whole group will asymptotically achieve the desired stable rigid motion. However, in practical situations, since it is required that all the agents attain a common velocity while maintaining the desired group shape, it is desired that $V_{ij}(t) \to 0$ and $x_{ij}(t) \to f_{ij}$ $\forall i,j \in V$, where f_{ij} is the desired distance between agents i and j in the goal topology. Furthermore, it is assumed that f_{ij} is also compatible in the sense that $f_{ik} + f_{kj} = f_{ij}$, $\forall i,j,k \in V$. Then the rigid flocking potential function V_{ij} should be devised for the edges in $E_{ij}(t) \cap E_g$ based on the following principles:

- V_{ij} is always nonnegative and differentiable in $(0, R_d)$; and
- V_{ij} attains its unique global minimum value when $\|x_{ij}\|$ obtains a predefined desired distance:

$$V_{ij}(\|x_{ij}\|) = \begin{cases} a \ln\|x_{ij}\|^2 + \frac{b}{\|x_{ij}\|^2} & 0 < \|x_{ij}\| \le \sqrt{\frac{b}{a}} \\ a \ln\|x_{ij}\|^2 + \frac{b}{\|x_{ij}\|^2} + \cos\left(1 + \frac{\|x_{ij}\|^2 - (b/a)}{R^2 - (b/a)}\right)\pi + 1 & \sqrt{\frac{b}{a}} < \|x_{ij}\| < R_d \\ a \ln\|x_{ij}\|^2 + \frac{b}{\|x_{ij}\|^2} + 2 & otherwise \end{cases} \quad (12)$$

where a, b and R_d are positive constants such that $b > (e/a)$, and $R_d > \sqrt{b/a}$. Note that the potential function V_{ij} is everywhere continuous differentiable in the domain (Figure 2), which could avoid the nonsmooth switching of the controllers brought by dynamical changing neighboring relations due to the motion of agents.

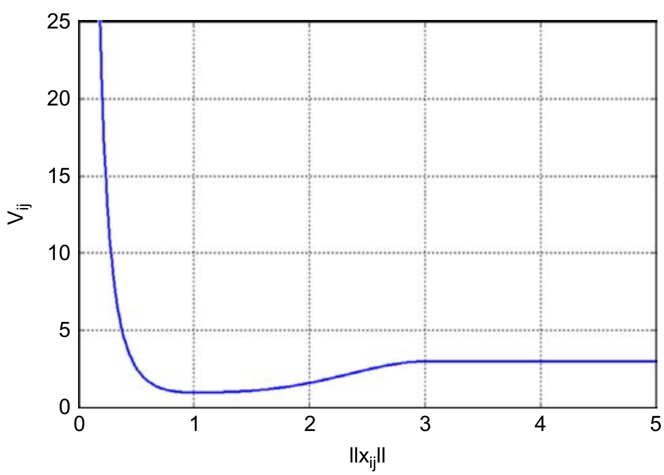


Figure 2.
Smooth potential function V_{ij} for a = 1

By the definition of V_{ij}, the total potential of agent i can be expressed as:

$$V_i = \sum_{j \notin N_i} V_{ij}(R) + \sum_{j \in N_i} V_{ij}(\|x_{ij}\|) \tag{13}$$

Note that during the course of motion, each agent regulates its position and velocity based on the external signal and the state information of its neighbors. However, it is known that, in reality, because of the influence of some external factors, the reference signal is not always detected by all agents in the group. In this paper, the case where the signal is sent continuously at any time is considered and we assume that there exists at least one agent in the group who can detect it.

The gradient control protocol from 12 for the stable infinitesimally rigid formation is in the following form:

$$u_i = \sum_{j \in N_i} -\nabla_{x_i} V_{ij} - \sum_{j \in N_i} w_{ij}(v_i - v_j) \tag{14}$$

where $v_l \in R^n$ is the desired common velocity and is a constant vector, $w_{ij} \geq 0, w_{ij} = w_{ji}$ and $w_{ii} = 0, i, j = 1, 2, \ldots, N$ represent the interaction coefficients.

4. Simulations and experiments

In this section, the experimental set-up and the results of flocking with real mobile robots in rigid formation are presented. The platform used in our experiment is the Pioneer 3 mobile robots and Amigobots which are differential-drive mobile robots with a unicycle-like kinematics. The PID controllers of the wheels velocity are developed by the manufacture. The program of the robots is implemented in C++ and runs in real-time on the robot's onboard computer. The assumptions are given as follows.

The following parameters were empirically chosen based on both the system dynamics and simulation results, while the fine-tuning was performed by trial and error. The multi-robot systems consist of two Pioneer 3-AT and three Amigobots wheeled mobile robots, the control period $\Delta t = 0.5$ s and the communication radius are uniformly set to $R = 2$ m. Corresponding parameters for connectivity-preserving artificial potentials and leader-follower potentials are $c_1 = 2, c_2 = 1, c = 0$ and $\mathrm{k} = 10$, respectively. Different from the simulations in Matlab, relative distance to nearby vehicles and relative heading are considered here. The linear and angular velocities of the robots are randomly chosen in the range of [0,1]m/s and [0,1]rad/s. Moreover, the encoder resolution is such that a quantization of 0.6 cm/s and 6 deg/s, respectively. All the simulations and experiments are conducted for 2-D flocking in real time simulation software MobileSim and real indoor environment.

The snapshots of three robots with various kinds of color to reach the target rigid formation are shown in Figure 3. For the former, the initial positions and headings of the group are shown in Figure 3(a). Figure 3(b) demonstrates the aggregation process of the swarm forced by the attraction/repulsion potentials which are mentioned in equation (12). Figure 3(c) and (d) shows the stable states of the system in steady state, and the three robots maintain the rigid formation moving. There are three kinds of color with robots trajectory in the snapshots and the process for this experiment is represented obviously. The worst-case rigidity index (w.r.i.) is adopted to measure the rigidity of formation (Zhu and Hu, 2009). When the value of w.r.i. is 0, the formation is not rigid and the value of w.r.i of the rigid formation must be more than 0. Larger the value is, more rigid the formation is.

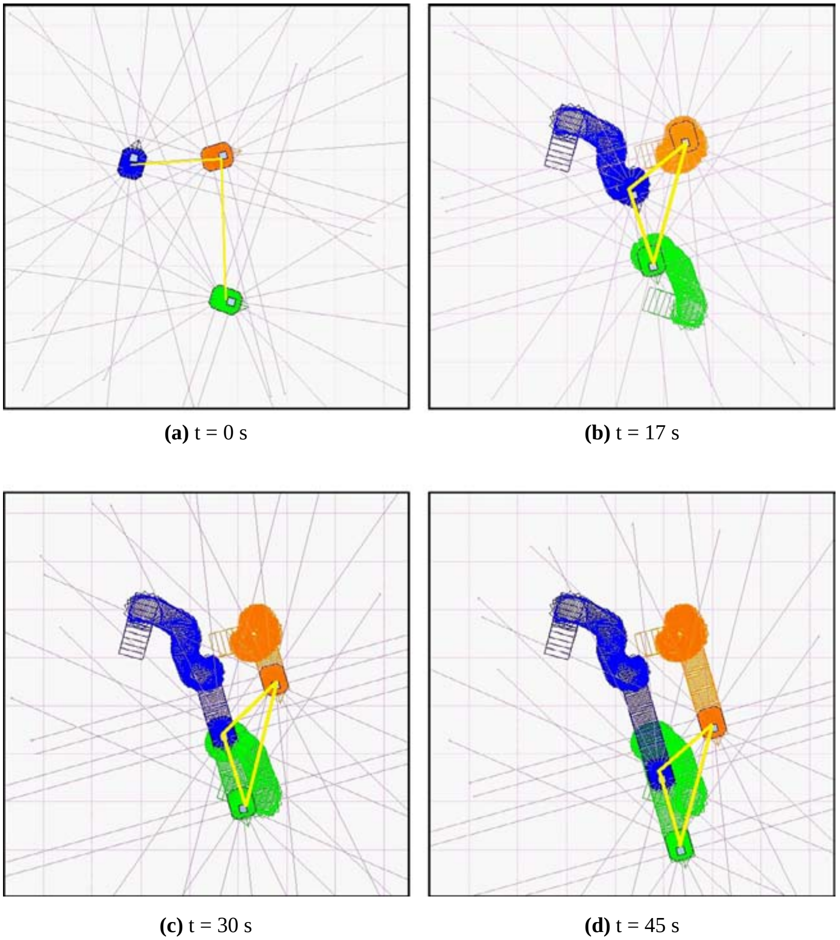


Figure 3.
Simulation snapshots for flocking of three wheeled mobile robots

Table I gives the worst-case rigidity index of formations in Figure 3. We see that Figure 1(a) is not rigid, but the others all keep the status of rigidity. It is obvious that there robots are easy to form the rigidly target formation, so the values of w.r.i. of Figure 1(b)-(d) are the same. However, it is different in Figures 4 and 5.

In a similar way, Figure 4(a)-(d) shows four typical snapshots of the system's formation evolution with five robots, and Table II is the content of the worst-case rigidity

Table I.
Rigidity indices of formations in Figure 3

Formation	No. of edges	w.r.i	Is rigid?
a	2	0	N
b	3	1.2	Y
c	3	1.2	Y
d	3	1.2	Y

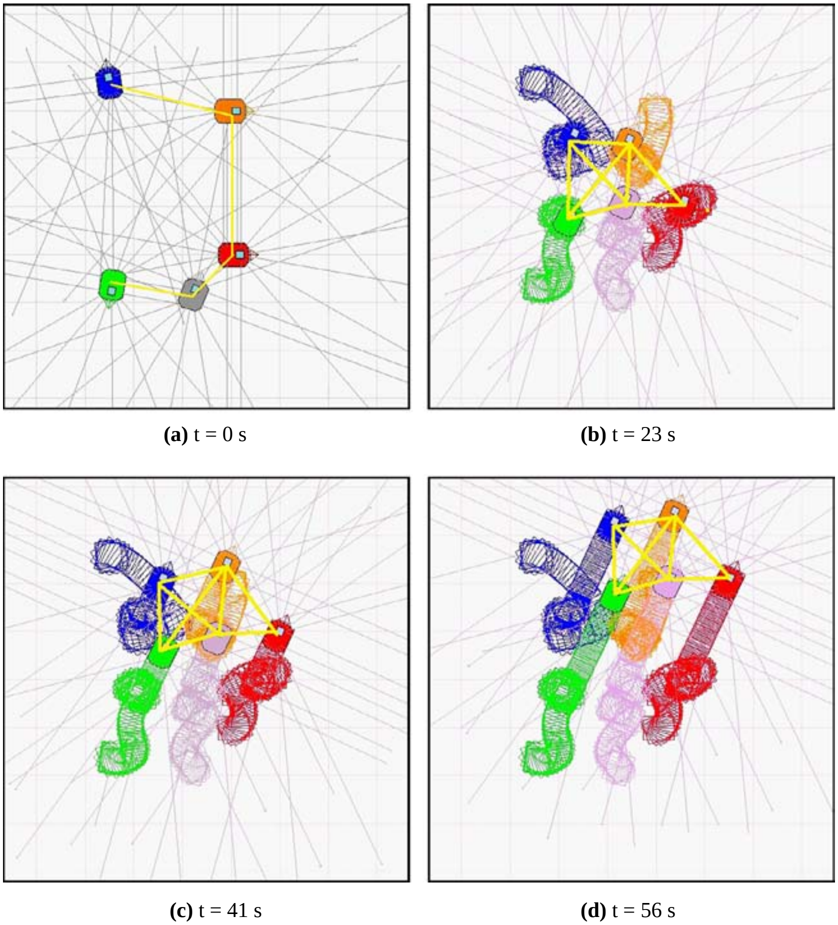
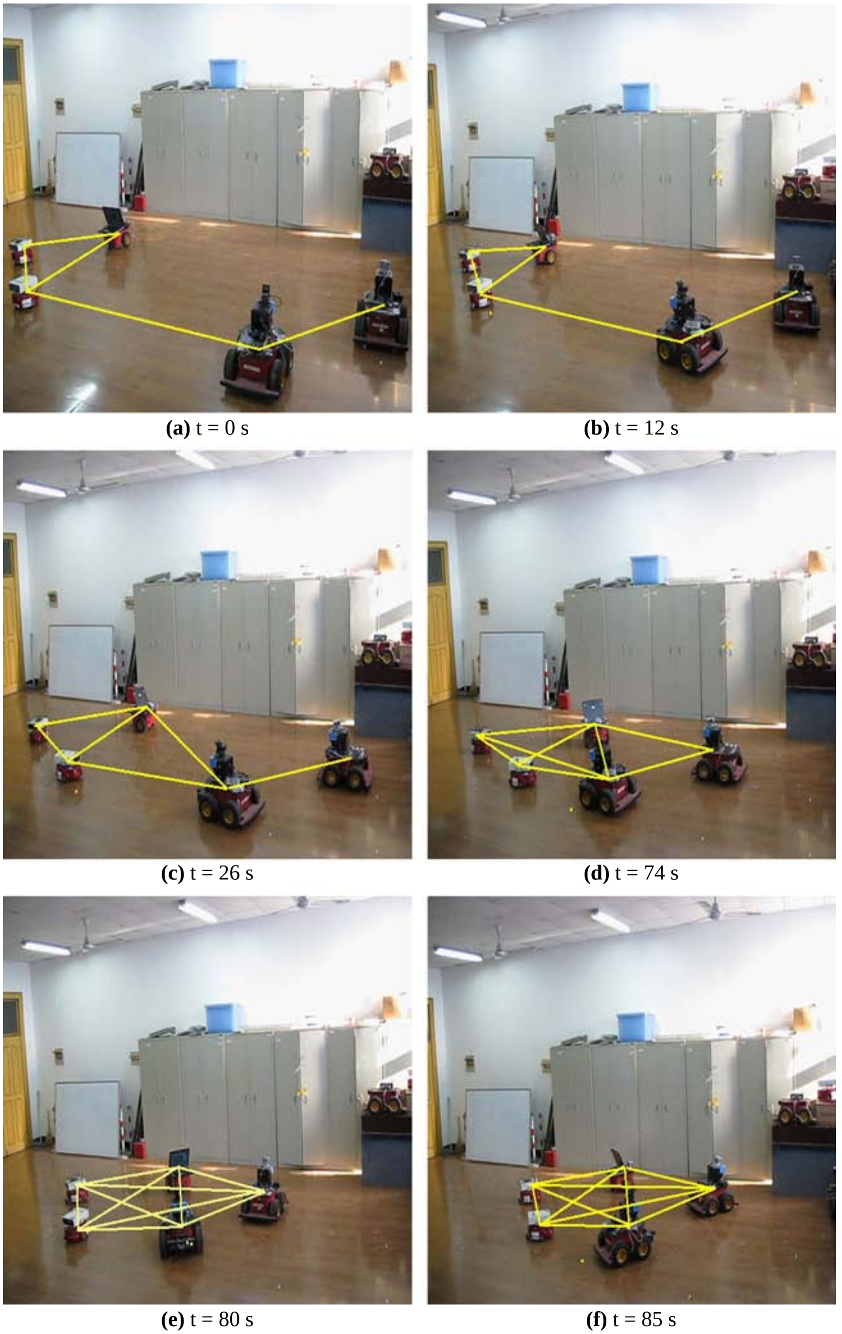


Figure 4.
Simulation snapshots for flocking of five wheeled mobile robots

index of formations in Figure 4. It is shown that the formation in Figure 4(b) is less rigid than Figure 4(c) and (d) as it is less symmetric.

From the pictures it is apparent that the velocity synchronization of the robots can be achieved, the distances between the interconnected robots are stabilized. Hence the group finally exhibits the rigid flocking motion.

Corresponding video snaps of the flocking experimental results with a period of 85 s are shown in Figure 5. Suppose that the start time $t_0 = 0s$. The initial state of the system is shown in Figure 5(a), in which the robots are in a stochastic formation which satisfies that the initial communication topology is connected. Figure 5(b) and (c) shows two typical snapshots which exhibit that the robots recover to form a cohesive group due to the potential function (12); Finally, the rigid formation of the swarm is shown in Figure 5(d) at $t = 74$ s, and Figure 5(e) and (f) shows that the five robots maintain the rigid formation for 11 s. It is clear that the experiments are well consistent with the corresponding simulation results.



(a) t = 0 s **(b)** t = 12 s **(c)** t = 26 s **(d)** t = 74 s **(e)** t = 80 s **(f)** t = 85 s

Figure 5.
Flocking of five wheeled mobile robots in indoor environment

Comparing formation Figure 5(a)-(f) from Table III, it is clearly seen that with the increased number of distance constraints, the value of worst-case rigidity index rises accordingly, so the formation become more rigid with the moving by the potential function control. Also, by the same distance constraints the symmetric formation is more rigid than the asymmetric and it is shown in Figure 5(e) and (f).

5. Conclusions

This paper studies the formation stabilization problem for multi-robot systems and only started on what is likely to be a fairly long road. A control algorithm to solve the formation rigidity problem for nearly arbitrary formations is proposed. The strong relationship between the stability of the target formation and the gradient control protocol are utilized to design the control laws which can be proved to make the target formation stable. Experimental validation of the undirected formation control is shown on a multi-vehicle system. These experiments express that the formation control is able to retain the rigidity formation. However, in fact there is a broader list of issues that need to be addressed in the future and we record some as follows:

- The rigidity matrix is not utilized in the design of control law. We would like to deal with the formation problem with the rigidity matrix and design the linear or nonlinear laws, with the rigidity matrix varying or not in the course of the motion.
- The worst-case rigidity index (w.r.i.) is an important tool for quantitative measure of formation rigidity, and it can only characterize the rigidity of a static formation. We wish to know if we want to measure the formation rigidity continuously, what can be done to modify the w.r.i.
- In simulations and experiments, we use at most five robots here. If more robots are provided, whether the control laws in this paper are effective and if not, how to modify.

Formation	No. of edges	w.r.i	Is rigid?
a	5	0	N
b	8	0.6116	Y
c	8	0.9889	Y
d	8	0.9889	Y

Table II. Rigidity indices of formations in Figure 4

Formation	No. of edges	w.r.i	Is rigid?
a	5	0	N
b	5	0	N
c	6	0	N
d	8	0.2583	Y
e	10	1.6429	Y
f	10	2.5	Y

Table III. Rigidity indices of formations in Figure 5

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