

Adaptive Robust Control of Servo Mechanisms With Compensation for Nonlinearly Parameterized Dynamic Friction

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Abstract—In this brief, an adaptive robust control (ARC) scheme with compensation for nonlinearly parameterized dynamic friction is proposed. Both parametric uncertainties and external disturbances are considered in this method. Our method takes advantage of a Lipschitzian property with respect to the parameters of nonlinearly parameterized model in the ARC design. The outcome is that the number of parameters to be updated in the ARC is equal to the number of unknown parameters in the plant, and thus the resulting control algorithm is convenient to be implemented. We have proved theoretically that the proposed method can not only guarantee desired transient performance for the system, but also make the magnitude of steady-state tracking error to be arbitrarily small in the presence of parametric uncertainties only. Experimental results are given to demonstrate the effectiveness of the proposed ARC scheme.

Index Terms—Adaptive robust control (ARC), friction compensation, servo mechanism.

I. INTRODUCTION

IN SERVO mechanisms, friction may cause many undesired phenomena such as large tracking errors, limit cycles, and stick-slip motion. Accordingly, it is important to compensate for the effects of friction, when high performance is needed for servo mechanisms. Many methods have been proposed to solve the friction compensation problem [1], [2]. Some control schemes (e.g., in [3], [4]) are based on an accurate offline friction estimation. The main drawback of this kind of methods is that, their design procedures need accurate models of friction, which are difficult to acquire. To overcome this problem, adaptive friction compensation techniques based on different friction models have been proposed in the literature [5]–[7]. In most of these results, friction is modeled as a static map between velocity and friction. However, in applications with high precision positioning and with low velocity tracking, friction compensation based on static models is not always satisfactory.

Several behaviors of friction, such as presliding displacement, hysteresis, friction lag, and etc., cannot be represented by the static models. To capture these effects, researchers have proposed a number of dynamic friction models [8]. For example, Dahl [9] proposed a dynamic model to capture the spring-like behavior during stiction. In [10], a dynamic friction

model called LuGre model is proposed. This model captures various friction effects such as the Stribeck effect, hysteresis, presliding displacement and varying break-away force.

Based on the LuGre model, several model-based controllers have been developed. In [10], a model-based controller with a friction state observer was proposed, under the assumption that the system parameters are exactly known. However, it is usually difficult to capture the nonlinear features of friction exactly, since every physical system is subject to certain degrees of model uncertainties. To account for the uncertainties in the LuGre model, some adaptive controllers have been presented. Two globally stable model-based adaptive friction compensation schemes were proposed in [11] to address parameter changes associated with normal force variation or temperature variation. In [12], an adaptive controller utilizing a nonlinear observer/filter structure was proposed to handle non-uniform variations in the friction force by assuming independent coefficient change as temperature varies. In [13], an adaptive controller is developed by introducing a nonlinear observer, which is utilized to estimate the unmeasurable friction state. In [14], the neural network is used to parameterize the unknown dynamic friction bounding function, and then an adaptive neural network controller is developed. However, these adaptive controllers suffer from two main drawbacks—unknown transient performance and possible non-robustness to disturbances.

To overcome these drawbacks, a dynamic friction compensation strategy is proposed in [15] by utilizing the idea of adaptive robust control (ARC) [16], [17], where the robust control term is used to guarantee desired transient response, and the adaptive control term is used to achieve favorable steady-state tracking accuracy. In this method, the parameters related to the Stribeck effect are assumed to be known. With these known parameters, a dual-observer is constructed to estimate the unknown friction states. A similar result is proposed in [18]. However, different from the method in [15], a modified LuGre model is adopted in [18], and thus the digital implementation problems of the LuGre-model-based dynamic friction compensation can be avoided. However, the above adaptive robust controllers are based on the assumption of known Stribeck curves, which is not easy to obtain.

In this brief, a novel ARC-based dynamic friction compensation strategy is proposed. This method does not need previously known Stribeck-effect-related parameters in its design procedure. Thus, it is more convenient for engineering application than the adaptive robust controllers proposed in [15] and [18]. Unlike the adaptive neural network friction compensator proposed in [14], the friction compensator is synthesized directly according to the nonlinearly parameterized model in our method. Hence, the number of parameters to be estimated is equal to the number of unknown parameters of the plant, which

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is much less than the parameter number of the adaptive neural network controller in [14]. Motivated by the recently proposed method in [19] and [20], our method takes advantage of a Lipschitzian property with respect to the parameters of nonlinearly parameterized model in the ARC design. Unlike [19] and [20], where the friction compensation is based on Stribeck model and only the parametric uncertainty is considered, this brief focuses on the adaptive robust control with friction compensation based on LuGre model for a DC servomechanism subjected to parametric and nonlinear uncertainties.

This brief is organized as follows. The problem description is presented in Section II. The design procedure of the proposed ARC is provided in Section III. The stability proof and performance analysis are given in Section IV. Experimental results are described and analyzed in Section V and conclusions are drawn in Section VI.

II. DYNAMIC MODELS AND PROBLEM FORMULATIONS

A. Dynamic Model of Servo Mechanisms

By ignoring the current dynamics, the mechanical and electrical dynamics of a current-controlled DC servo mechanism can be described as [21]

$$J\ddot{q} = T_m - T_f - T_l - T_{\text{dis}} \quad (1)$$

$$T_m = K_I u_I \quad (2)$$

where q is the angular position; u_I is the current input command; T_m , T_f , T_l , and T_{dis} represent the torques imposed on the motor shaft: generated torque, friction torque, load torque, and disturbance torque, respectively; J represents the total motor and load inertia; K_I denotes the electrical-mechanical energy conversion constant. Using the dynamic friction model proposed in [10], the dynamic friction torque T_f can be described by

$$T_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{q} \quad (3)$$

$$\dot{z} = \dot{q} - \sigma_0 \frac{|\dot{q}|}{g(\dot{q})} z \quad (4)$$

where z represents the unmeasured internal friction state; σ_0 , σ_1 , and σ_2 are positive parameters that can be physically explained as the stiffness of bristles, damping coefficient associated with \dot{z} , and viscous coefficient. The function $g(\dot{q})$ is utilized to describe the Stribeck effect [10] and given as follows:

$$g(\dot{q}) = T_c + (T_s - T_c)e^{-(\dot{q}/\omega_s)^2} \quad (5)$$

where ω_s is the Stribeck velocity; T_c and T_s represent the levels of Coulomb friction and stiction, respectively. In general, $0 < T_c \leq T_s$, thus, (5) implies that $T_c \leq g(\dot{q}) \leq T_s$. The friction model given by (3)–(5) reflects the dynamic effects caused by the deflection of bristles, which are used to model the asperities between two contacting surfaces. It is shown in [11] and [22] that the above friction model has the following finite bristle deflection property.

Property 1: Under the assumption that $0 < z_{\min} \leq (g(\dot{q})/\sigma_0) \leq z_{\max}$, we have, if $|z(0)| \leq z_{\max}$, then $|z(t)| \leq z_{\max}, \forall t \geq 0$.

This property will be used in the subsequent controller design. Here, all the parameters of the plant (1)–(5) are assumed to be unknown. Thus, it can be concluded that the plant studied in this brief is subject to dynamic friction, parametric uncertainties and external disturbance. The angular position q is regarded as the system output. Defining the angular position and angular velocity as the state variables, i.e., $[x_1, x_2]^T = [q, \dot{q}]^T$, T_n as the mean value (direct offset) of $T_l + T_{\text{dis}}$ and d as the disturbance term, i.e., $d = T_n - T_l - T_{\text{dis}}$, from (1)–(5), the entire system can be expressed in the state space form as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \alpha_1 u_I - \alpha_2 x_2 - \alpha_3 - T_{fn}(x_2, \sigma_0 z, \beta) + \Delta \\ \dot{z} = x_2 - \sigma_0 \frac{|x_2|}{g(x_2)} z \\ y = x_1 \\ T_{fn}(x_2, \sigma_0 z, \beta) = \sigma_0 z \left[\beta_1 \left(1 - \frac{\beta_2 |x_2|}{1 + (\beta_3 - 1)e^{-\beta_4 x_2^2}} \right) \right] \end{cases} \quad (6)$$

where $\Delta = d/J$ denotes the lumped disturbance; $\alpha := [\alpha_1, \alpha_2, \alpha_3]^T = [K_I/J, (\sigma_1 + \sigma_2)/J, T_n/J]^T$ is the parameter vector consisting of the linearly parameterized coefficients; $\beta := [\beta_1, \beta_2, \beta_3, \beta_4]^T = [1/J, \sigma_1/T_c, T_s/T_c, 1/\omega_s^2]^T$ is the parameter vector consisting of the nonlinearly parameterized coefficients.

B. Assumptions and Problem Formulations

For simplicity, the following notations will be used: \bullet_i for the i th component of the vector \bullet , $\hat{\bullet}$ for the estimate of \bullet , \bullet_{\min} for the minimum value of \bullet , and \bullet_{\max} for the maximum value of \bullet . The operation \leq for two vectors is performed in terms of the corresponding elements of the vectors.

In this brief, we assume that the uncertain parameters are in certain known intervals, as shown in Assumptions 1 ~ 3. In addition, Assumption 4 is made for the desired motion trajectory $x_{1d}(t)$.

Assumption 1: $\alpha_i \in [\alpha_{i\min}, \alpha_{i\max}]$, $\beta_j \in [\beta_{j\min}, \beta_{j\max}]$ ($i = 1, \dots, 3, j = 1, \dots, 4$), where $\alpha_{i\min}$, $\alpha_{i\max}$, $\beta_{j\min}$, and $\beta_{j\max}$ are previously known positive numbers. In addition, it is assumed that $\alpha_{1\min} > 0$ and $\beta_{3\min} \geq 1$, which is coherent with the fact that $J > 0$ and $T_s \geq T_c$.

Assumption 2: The stiction is upper bounded, i.e., $T_s \leq T_{\max}$, where T_{\max} is known.

Assumption 3: The disturbance Δ is bounded, i.e., $|\Delta| \leq \delta$, where δ is known.

Assumption 4: The desired trajectory x_{1d} is continuous with first derivative \dot{x}_{1d} and second-order derivative \ddot{x}_{1d} available.

Consider model (6) which has dynamic friction nonlinearity, parametric uncertainties and disturbance. The control problem of this brief can be stated as follows. Given the desired motion trajectory $x_{1d}(t)$, the objective is to synthesize a control input u_I such that the system output $y = x_1$ tracks $x_{1d}(t)$ as closely as possible in spite of dynamic friction and various model uncertainties.

III. ADAPTIVE ROBUST CONTROL

In this section, we are going to detail the adaptive robust control design for the plant (6).

A. Discontinuous Control Law

The control law design follows the ARC synthesis procedure proposed in [16], [17] and [25]. First, define a tracking-error-index-like variable [26] e_2 as

$$e_2 = \dot{e}_1 + k_p e_1 = x_2 - x_{2eq}, \quad x_{2eq} := \dot{x}_{1d} - k_p e_1 \quad (7)$$

where $e_1 = x_1 - x_{1d}(t)$ is the output tracking error, and k_p is any positive feedback gain. Since $G_s(s) = e_1(s)/e_2(s) = 1/(s + k_p)$ is a stable transfer function, if e_2 is small or converges to zero exponentially, then the output tracking error e_1 will be small or converge to zero exponentially. Differentiating (7), from (6), we obtain

$$\begin{aligned} \dot{e}_2 &= \dot{x}_2 - \dot{x}_{2eq} \\ &= \alpha_1 u_I - \alpha_2 x_2 - \alpha_3 - T_{fn}(x_2, \sigma_0 z, \beta) + \Delta - \dot{x}_{2eq} \end{aligned} \quad (8)$$

where $\dot{x}_{2eq} := \ddot{x}_{1d} - k_p \dot{e}_1$. Define a positive definite function V as $V(e_2) = (1/2)e_2^2$. Then, from (8), the derivative of V can be derived as

$$\dot{V} = e_2(\alpha_1 u_I - \alpha_2 x_2 - \alpha_3 + \Delta - \dot{x}_{2eq}) - e_2 T_{fn}(x_2, \sigma_0 z, \beta). \quad (9)$$

From the right-hand side of (9), we observe that a nonlinearly parameterized term $e_2 T_{fn}(x_2, \sigma_0 z, \beta)$ influences the derivative of V .

Viewing Property 1 and Assumption 2, we obtain the following result.

Lemma 1: Under Assumption 2, if $|\sigma_0 z(0)| \leq T_{\max}$, then $|\sigma_0 z(t)| \leq T_{\max}$, $\forall t \geq 0$.

Proof: Noting that $0 < T_c \leq g(x_2) \leq T_s$, from Assumption 2, we get $g(x_2)/\sigma_0 \leq T_{\max}/\sigma_0$. Combining with Property 1, it can be concluded that $|z(t)| \leq T_{\max}/\sigma_0$, $\forall t \geq 0$, if $|z(0)| \leq T_{\max}/\sigma_0$, which yields the result of Lemma 1.

From Lemma 1, one has

$$\begin{aligned} -e_2 T_{fn}(x_2, \sigma_0 z, \beta) \\ \leq e_2 \text{sgn}(e_2) T_{\max} \beta_1 \left[1 + \frac{\beta_2 |x_2|}{1 + (\beta_3 - 1)e^{-\beta_4 x_2^2}} \right]. \end{aligned} \quad (10)$$

Define a nonlinearly parameterized function $f(x_2, \beta)$ as

$$f(x_2, \beta) = \beta_1 \left[1 + \frac{\beta_2 |x_2|}{1 + (\beta_3 - 1)e^{-\beta_4 x_2^2}} \right]. \quad (11)$$

To find the bound of $f(x_2, \beta)$, the definition of Lipschitzian condition is recalled first.

Definition 1: The function $f(x_2, \beta)$ is said to be locally Lipschitzian in β if there exist continuous functions $L_i(x_2)$, $i = 1, \dots, 4$ such that the following inequality holds:

$$|f(x_2, \beta') - f(x_2, \beta'')| \leq \sum_{i=1}^4 L_i(x_2) |\beta'_i - \beta''_i|$$

$$\forall \beta', \beta'' \in \Omega_\beta := \{\beta | \beta_{i \min} \leq \beta_i \leq \beta_{i \max}, i = 1, \dots, 4\} \quad (12)$$

where $\beta' = [\beta'_1, \beta'_2, \beta'_3, \beta'_4]^T$, $\beta'' = [\beta''_1, \beta''_2, \beta''_3, \beta''_4]^T$.

Lemma 2: The function $f(x_2, \beta)$ is Lipschitzian in β if $\partial f(x_2, \beta)/\partial \beta$ exists and is continuous in $R \times \Omega_\beta$, furthermore, the continuous functions $L_i(x_2)$, $i = 1, \dots, 4$ in (12) can be chosen as any continuous function satisfying $L_i(x_2) \geq \max_{\beta \in \Omega_\beta} |\partial f(x_2, \beta)/\partial \beta_i|$.

Proof: See Appendix A.

Then, from Lemma 2, we know that $f(x_2, \beta)$ given by (11) is locally Lipschitzian in β , because $\partial f(x_2, \beta)/\partial \beta$ exists and is continuous in $R \times \Omega_\beta$. Combining Lemma 2 with (11), and noting Assumption 1, it is easy to check that one choice of $L_i(x_2)$, $i = 1, \dots, 4$ is

$$\begin{cases} L_1(x_2) = 1 + \frac{\beta_2 \max x_{2b}}{1 + (\beta_3 \min - 1)e^{-\beta_4 \max x_2^2}} \\ L_2(x_2) = \frac{\beta_1 \max x_{2b}}{1 + (\beta_3 \min - 1)e^{-\beta_4 \max x_2^2}} \\ L_3(x_2) = \frac{\beta_1 \max \beta_2 \max x_{2b} e^{-\beta_4 \min x_2^2}}{\left[1 + (\beta_3 \min - 1)e^{-\beta_4 \max x_2^2} \right]^2} \\ L_4(x_2) = \frac{\beta_1 \max \beta_2 \max (\beta_3 \max - 1) x_{2b} x_2^2 e^{-\beta_4 \min x_2^2}}{\left[1 + (\beta_3 \min - 1)e^{-\beta_4 \max x_2^2} \right]^2} \end{cases} \quad (13)$$

where $x_{2b} = (1/4b)x_2^2 + b$ with $b > 0$. In view of (10)–(13), it follows that

$$\begin{aligned} -e_2 T_{fn}(x_2, \sigma_0 z, \beta) \\ \leq e_2 \text{sgn}(e_2) T_{\max} L^T(x_2) \gamma - e_2 T_{fn}(x_2, \sigma_0 z, \beta_{\min}) \\ \leq e_2 \text{sgn}(e_2) T_{\max} L^T(x_2) \gamma \\ + e_2 \text{sgn}(e_2) T_{\max} f(x_2, \beta_{\min}) \end{aligned} \quad (14)$$

where $L(x_2) := [L_1(x_2), L_2(x_2), L_3(x_2), L_4(x_2)]^T$ and γ is defined as

$$\gamma = \beta - \beta_{\min}. \quad (15)$$

According to Assumption 1, we know that $\gamma_{\min} \leq \gamma \leq \gamma_{\max}$, where $\gamma_{\min} = 0$ and $\gamma_{\max} = \beta_{\max} - \beta_{\min}$. For the convenience of deduction, we define the parameter vector θ as $\theta := [\alpha^T, \gamma^T]^T \in R^7$. Then, it is easy to check that the upper and lower bounds of θ are $\theta_{\min} = [\alpha_{\min}^T, \gamma_{\min}^T]^T$ and $\theta_{\max} = [\alpha_{\max}^T, \gamma_{\max}^T]^T$.

Viewing (9), (14), and (15), we obtain

$$\begin{aligned} \dot{V} &\leq e_2 [\alpha_1 u_I - \alpha_2 x_2 - \alpha_3 + \text{sgn}(e_2) T_{\max} L^T(x_2) \gamma \\ &\quad - \dot{x}_{2eq} + \text{sgn}(e_2) T_{\max} f(x_2, \beta_{\min}) + \Delta]. \end{aligned} \quad (16)$$

According to (16), the control law can be synthesized by using the ARC approach. The control input u_I is designed as

$$\begin{cases} u_I = u_a + u_s \\ u_a = \frac{1}{\hat{\alpha}_1} [\hat{\alpha}_2 x_2 + \hat{\alpha}_3 - \text{sgn}(e_2) T_{\max} L^T(x_2) \hat{\gamma} + \dot{x}_{2eq} \\ \quad - \text{sgn}(e_2) T_{\max} f(x_2, \beta_{\min})] \\ u_s = u_{s1} + u_{s2}, u_{s1} = -\frac{k_s}{\alpha_{1 \min}} e_2 \end{cases} \quad (17)$$

where $\hat{\alpha}_i$ is the estimate of α_i , $i = 1, \dots, 4$; $\hat{\gamma}$ is the estimate of γ ; $k_{s1} > 0$ is a controller parameter to be chosen; u_a is an adaptive control term; u_s is a robust control term consisting of two parts, i.e., u_{s1} and u_{s2} . The robust control function u_{s2} should be selected to satisfy

$$e_2 u_{s2} \leq 0 \quad (18)$$

$$e_2 [\alpha_1 u_{s2} - \varphi_0^T \hat{\theta} + \Delta] \leq \varepsilon_0 \quad (19)$$

where $\varphi_0^T = [u_a, -x_2, -1, \text{sgn}(e_2)T_{\max}L^T(x_2)]^T$ and $\varepsilon_0 > 0$ is a design parameter. The parameter is updated by

$$\dot{\hat{\theta}} = \text{Proj}_{\hat{\theta}}(\Gamma\tau), \quad \Gamma > 0 \quad (20)$$

where $\text{Proj}(\bullet)$ represents the projection operation whose definition can be seen in [27], the positive definite matrix Γ represents the adaptation rate, the adaptation function τ is synthesized as $\tau = \varphi_0 e_2$. Since $\alpha_1 \geq \alpha_{1\min} > 0$ (as is indicated in Assumption 1) and the projection operator is used in the adaptation law to guarantee $\theta_{\min} \leq \hat{\theta} \leq \theta_{\max}$, we know that there must be certain robust control function u_{s2} satisfying (18) and (19). There are many ways to design u_{s2} . One alternative is to let

$$u_{s2} = -\frac{h_0}{2\alpha_{1\min}\varepsilon_0}e_2$$

where h_0 is any function or constant satisfying $h_0 \geq \|\varphi_0\|^2 \|\theta_{\max} - \theta_{\min}\|^2 + \delta^2$. Readers can refer to [16] and [24] for other choices of u_{s2} .

Lemma 3: Suppose that the control law (17) and the adaptation law (20) are applied to control the plant (6), which satisfies Assumptions 1 ~ 4. Then, no matter what the adaptation function τ is, the controller guarantees that: The closed-loop system is globally stable, and the positive definite function V defined by $V = (1/2)e_2^2$ is bounded above by

$$V \leq \exp(-\lambda t)V(0) + \frac{\varepsilon_0}{\lambda}(1 - \exp(-\lambda t)) \quad (21)$$

where $\lambda = 2k_{s1}$.

Proof: Substituting (17) into (16), we have

$$\begin{aligned} \dot{V} &\leq -k_{s1}e_2^2 \\ &\quad + e_2 [\alpha_1(u_a + u_{s2}) - \alpha_2x_2 - \alpha_3 - \dot{x}_{2eq} \\ &\quad \quad + \text{sgn}(e_2)T_{\max}(L^T(x_2)\gamma + f(x_2, \beta_{\min})) + \Delta] \\ &= -k_{s1}e_2^2 + e_2 [\alpha_1 u_{s2} - \tilde{\alpha}_1 u_a + \tilde{\alpha}_2 x_2 + \tilde{\alpha}_3 \\ &\quad \quad - \text{sgn}(e_2)T_{\max}L^T(x_2)\tilde{\gamma} + \Delta]. \end{aligned} \quad (22)$$

Combining with (19), it follows that

$$\dot{V} \leq -\lambda V + \varepsilon_0 \quad (23)$$

which implies (21).

Remark 1: Since $V = (1/2)e_2^2$ is nonincreasing and bounded from below by zero, according to the inequality (21), one has

$$\left(\frac{1}{2}e_2^2(t)\right) \leq -\frac{1}{2}\exp(-\lambda t)|e_2(0)|^2 + \frac{\varepsilon_0}{\lambda}(1 - \exp(-\lambda t)).$$

It indicates that the tracking error $e_2(t)$ will exponentially converge to a bounded value ε_0/λ . There is no overshoot and the convergent rate can be prescribed by tuning parameter λ

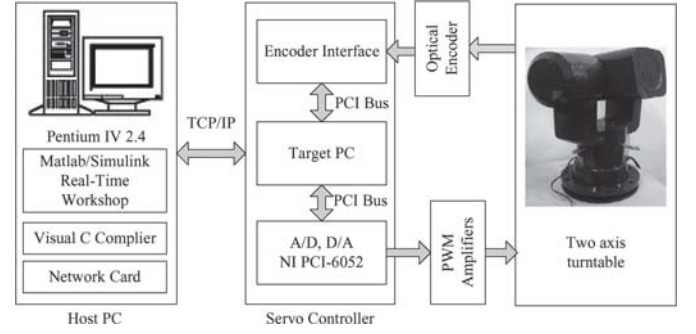


Fig. 1. Structure of the two-axis servo system.

which is determined by control gain. Moreover, in order to obtain better final tracking accuracy, smaller ε_0 or larger λ is required which can be realized only by choosing large robust control gain when the parameter adaptation is closed. However, too large gain will lead the controller to be saturated. Thus, activating the adaptation mechanism is very helpful to reduce the conservatism of the controller.

Remark 2: Lemma 3 shows that the discontinuous control law can not only guarantee the closed-loop system to be stable, but also make the tracking error converge to a small prescribed region. However, due to the discontinuity at $e_2 = 0$ caused by the term $\text{sgn}(e_2)$, the closed-loop system controlled by the control law (17) may suffer from chattering, which is not desired in the application. To solve this problem, in the next subsection, we will do some modification to the above control law design so that a continuous control action can be obtained.

B. Continuous Control Law

A continuous control law can be derived by modifying the tracking-error-index-like variable. Here, we introduce a new variable e_ε as follows [19]:

$$e_\varepsilon = e_2 - \frac{1}{\sqrt{3}}c(e_2) \quad (24)$$

where

$$c(e_2) = \begin{cases} d_1 + \sqrt{d_2^2 - (e_2 - \varepsilon)^2}, & \frac{\sqrt{3}-1}{2}\varepsilon \leq e_2 \leq \varepsilon \\ \sqrt{3}e_2, & |e_2| \leq \frac{\sqrt{3}-1}{2}\varepsilon \\ -d_1 - \sqrt{d_2^2 - (e_2 + \varepsilon)^2}, & -\varepsilon \leq e_2 \leq \frac{1-\sqrt{3}}{2}\varepsilon \\ \varepsilon \text{sgn}(e_2), & |e_2| > \varepsilon \end{cases} \quad (25)$$

with $d_1 = (2 - \sqrt{3})\varepsilon$, $d_2 = (\sqrt{3} - 1)\varepsilon$, for $\varepsilon > 0$. It has been indicated in [19] that e_ε is a continuously differentiable function of e_2 (see Fig. 1). From (25), e_ε/e_2 can be derived as

$$\frac{\partial e_\varepsilon}{\partial e_2} = \begin{cases} 1 + \frac{e_2 - \varepsilon}{\sqrt{3[d_2^2 - (e_2 - \varepsilon)^2]}}, & \frac{\sqrt{3}-1}{2}\varepsilon \leq e_2 \leq \varepsilon \\ 0, & |e_2| \leq \frac{\sqrt{3}-1}{2}\varepsilon \\ 1 - \frac{e_2 + \varepsilon}{\sqrt{3[d_2^2 - (e_2 + \varepsilon)^2]}}, & -\varepsilon \leq e_2 \leq \frac{1-\sqrt{3}}{2}\varepsilon \\ 1, & |e_2| > \varepsilon. \end{cases} \quad (26)$$

Define a positive definite function V_ε as $V_\varepsilon(e_\varepsilon) = (1/2)e_\varepsilon^2$. Then, from (8), it follows that

$$\begin{aligned} \dot{V}_\varepsilon &= e_\varepsilon \frac{\partial e_\varepsilon}{\partial e_2} [\alpha_1 u_I - \alpha_2 x_2 - \alpha_3 \\ &\quad - T_{fn}(x_2, \sigma_0 z, \beta) + \Delta - \dot{x}_{2eq}]. \end{aligned} \quad (27)$$

The (25) and (26) imply that, for any $|e_\varepsilon| > 0$, we have $e_\varepsilon/e_2 > 0$. Hence, to make e_ε converge to zero, we only need to let $\alpha_1 u_I - \alpha_2 x_2 - \alpha_3 - T_{fn}(x_2, \sigma_0 z, \beta) + \Delta - \dot{x}_{2eq} < 0$. Similar to (14), it can be deduced that

$$\begin{aligned} & -e_\varepsilon T_{fn}(x_2, \sigma_0 z, \beta) \\ & \leq |e_\varepsilon| T_{\max} L^T(x_2)(\beta - \beta_{\min}) - e_\varepsilon T_{fn}(x_2, \sigma_0 z, \beta_{\min}) \\ & \leq |e_\varepsilon| T_{\max} L^T(x_2)\gamma + |e_\varepsilon| T_{\max} f(x_2, \beta_{\min}). \end{aligned} \quad (28)$$

From the definition of e_ε , it is easy to verify that

$$|e_\varepsilon| = e_\varepsilon \sigma_\varepsilon(e_2) \quad (29)$$

where $\sigma_\varepsilon(e_2)$ is a continuous function of e_2 and defined as

$$\sigma_\varepsilon(e_2) = \begin{cases} 1 & e_2 > (\sqrt{3} - 1)\varepsilon/2 \\ 2e_2/(\sqrt{3}\varepsilon - \varepsilon) & |e_2| \leq (\sqrt{3} - 1)\varepsilon/2 \\ -1 & e_2 < (1 - \sqrt{3})\varepsilon/2. \end{cases}$$

Combining (28) and (29) with (27), one has

$$\begin{aligned} \dot{V}_\varepsilon \leq e_\varepsilon \frac{\partial e_\varepsilon}{\partial e_2} & [\alpha_1 u_I - \alpha_2 x_2 - \alpha_3 + \sigma_\varepsilon(e_2) T_{\max} L^T(x_2)\gamma \\ & + \sigma_\varepsilon(e_2) T_{\max} f(x_2, \beta_{\min}) + \Delta - \dot{x}_{2eq}]. \end{aligned} \quad (30)$$

Based on the above inequality, the continuous control law can be synthesized as

$$\begin{cases} u_I = u_a + u_s \\ u_a = \frac{1}{\alpha_1} [\hat{\alpha}_2 x_2 + \hat{\alpha}_3 - \sigma_\varepsilon(e_2) T_{\max} L^T(x_2)\hat{\gamma} + \dot{x}_{2eq} \\ \quad - \sigma_\varepsilon(e_2) T_{\max} f(x_2, \beta_{\min})] \\ u_s = u_{s1} + u_{s2}, u_{s1} = -\frac{k_{s1}}{\alpha_1 \min} e_\varepsilon \end{cases} \quad (31)$$

The robust control function u_{s2} should be selected to satisfy

$$e_\varepsilon u_{s2} \leq 0 \quad (32)$$

$$e_\varepsilon [\alpha_1 u_{s2} - \varphi^T \tilde{\theta} + \Delta] \leq \varepsilon_1 \quad (33)$$

where $\varphi^T = [u_a, -x_2, -1, \sigma_\varepsilon(e_2) T_{\max} L^T(x_2)]^T$, and $\varepsilon_1 > 0$ is a design parameter which can be arbitrarily small. Similar to the discontinuous control law design, one alternative of u_{s2} is

$$u_{s2} = -h/(2\alpha_1 \min \varepsilon_1 e_\varepsilon)$$

where h is any function or constant satisfying $h \geq \|\varphi\|^2 \|\theta_{\max} - \theta_{\min}\|^2 + \delta^2$. Thus, the continuous control law can assure the closed-loop system stable if the adaption law (20) is applied with $\tau = e_\varepsilon (\partial e_\varepsilon / \partial e_2) \varphi$. The rigorous proof of this result will be given in the Section IV. It will be verified that, by utilizing the above adaptation law, the modified tracking-error-index-like variable e_ε can be steered to origin, in the presence of parametric uncertainties only.

IV. STABILITY PROOF AND PERFORMANCE ANALYSIS

To analyze the control performance, we define the following level set of the positive definite function V_ε , for any $\rho > 0$

$$L_{V_\varepsilon}(\rho) = \{e_\varepsilon : V_\varepsilon(e_\varepsilon) = (1/2)e_\varepsilon^2 \leq \rho\}. \quad (34)$$

In addition, a constant ε' is defined as

$$\varepsilon' = v_0 + \max \left((2 - \sqrt{3})\varepsilon^2/3, \quad \varepsilon_1/\lambda \right) \quad (35)$$

where v_0 is any positive number. It can be arbitrarily small.

Lemma 4: Suppose that the control law (31) and the adaptation law (20) are applied to control the plant (6), which satisfies Assumptions 1 ~ 4. Then, no matter what the adaptation function τ is, the controller guarantees the closed-loop system to be globally stable, and the trajectories starting outside $L_{V_\varepsilon}(\varepsilon')$ will enter $L_{V_\varepsilon}(\varepsilon')$ in a finite time and remain inside $L_{V_\varepsilon}(\varepsilon')$ thereafter; moreover, the trajectories starting from $L_{V_\varepsilon}(\varepsilon')$ will remain inside this level set.

Proof: See Appendix B.

Remark 3: From the proof of Lemma 4, we observe that, the tracking-error-index-like variable e_ε can be steered into a sufficiently small region by increasing the robust control gain. However, every practical system is subject to input saturation. To avoid too large control action, the robust control gain should not be set too large. In fact, the adaptive control term can further improve the accuracy of the servo system, which will be demonstrated in the following deduction.

Theorem 1: Under the assumptions 1 ~ 4, the closed-loop system comprised of the plant (6), the control law (31) and the adaptation law (20) has the following properties.

- 1) No matter what the adaptation function τ is, the tracking-error-index-like variable e_2 satisfies $|e_2(t)| \leq \sqrt{2\varepsilon'} + \varepsilon$, $\forall t \geq t_0, \exists t_0 > 0$, meaning that the tracking error will be within a prescribed bound after a finite time.
- 2) If $\Delta = 0$, i.e., in the presence of parametric uncertainties only, and the adaptation function τ is given by (20) with $\tau = e_\varepsilon (\partial e_\varepsilon / \partial e_2) \varphi$, then, in addition to results in i), we have $e_\varepsilon \rightarrow 0$ as $t \rightarrow \infty$, which implies that $|e_2(t)| \leq (\sqrt{3} - 1)\varepsilon/2$ as $t \rightarrow \infty$.

Proof: See Appendix C.

Remark 4: Theorem 1 shows that the tracking error bound of the closed-loop system with the adaptation function (20) is much smaller than that of the system without considering the adaptation law. Thus, the tracking accuracy is improved by parameter adaptation. On the other hand, due to the utilization of parameter adaptation, the system can achieve high accuracy without using large robust control gain.

V. EXPERIMENTAL RESULTS

In this section, experimental results are obtained from a DC torque motor servomechanism to demonstrate the effectiveness of the proposed method.

A. Experimental Setup

According to our previous work [27], a two-axis turntable servo system (see Fig. 1) is set up as a test-bed.

The position and velocity signals are collected by the optical encoder with 0.0005 degree resolution. The controller is implemented through an Xpc-Target that consists of a target personal computer and the interface card NI PCI-6052E. The sampling rate of the servo controller is 2 kHz. Experiments are con-

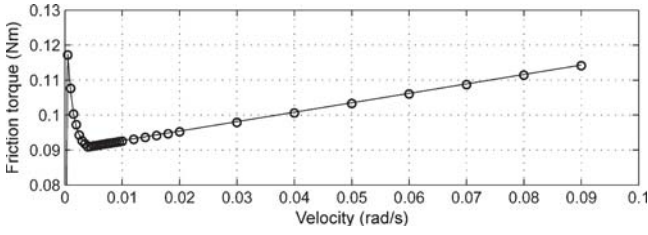


Fig. 2. Stribeck curve.

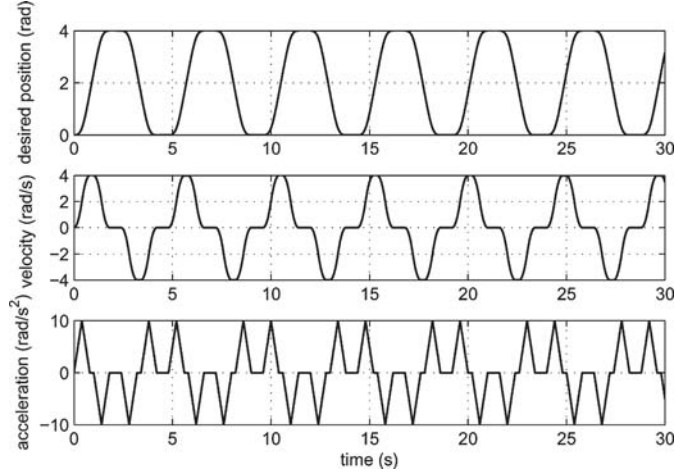


Fig. 3. Desired trajectory.

ducted on the pitch-axis which undertakes the payload (e.g., TV camera). The unbalance payload is considered as the external disturbance.

B. Determination of the Bounds of Unknown Parameters

From the (6), we know that the unknown parameters α_i, β_j are relative to K_I, J , average load torque T_n and parameters in friction model. From our previous work [27], we know $J = 0.011 \text{ kgm}^2$, $K_I = 6.36 \text{ N/A}$, $T_n = 0.1 \text{ Nm}$. To determine the friction parameters, offline parameter identification is carried out following the same procedure as in [10]. The Stribeck curve is shown in Fig. 2, from which we can roughly get $g(x_2) = 0.09171 + 0.0255e^{-(x/0.0477)^2} \text{ Nm}$ and $\sigma_2 = 0.2702 \text{ Nm/(rad/s)}$.

To obtain σ_0, σ_1 , let the system operate around zero velocity, give it a step input and measure the output response. With these experiments, finally, we get $\sigma_0 = 421.6 \text{ N}$ and $\sigma_1 = 6.738 \text{ Nm/(rad/s)}$. Consequently, we can get $T_c = 0.09171 \text{ N}$, $T_s = T_c + 0.0255 = 0.11721 \text{ N}$. The motor driver is set as torque mode. The gain of the pseudo amplifier is $k_a = 2.4$ obtained from the driver specification. Then noting (6), we know that $\alpha_1 = K_I k_a / J = 1387.63 \text{ (N/A)/(kgm}^2\text{)}$, $\alpha_2 = (\sigma_1 + \sigma_2) / J = 637.12 \text{ rad}^{-1}\text{s}^{-1}$, $\alpha_3 = T_n / J = 9.09 \text{ N/(kgm}^2\text{)}$, $\beta_1 = 1/J = 90.91 \text{ (kgm}^2\text{)}^{-1}$, $\beta_2 = \sigma_1 / T_c = 61.25 \text{ m/(rad/s)}$, $\beta_3 = T_s / T_c = 1.278$, $\beta_4 = 1/\omega_s^2 = 500 \text{ [(rad/s)}^2\text{)}^{-1}$. The bounds of the parameter variations in the experiment are chosen as $\alpha_{\min} = [1248, 573, 8]^T$, $\alpha_{\max} = [1526, 701, 10]^T$, $\beta_{\min} = [81, 55, 1.1, 450]^T$, $\beta_{\max} = [100, 68, 1.5, 550]^T$. To

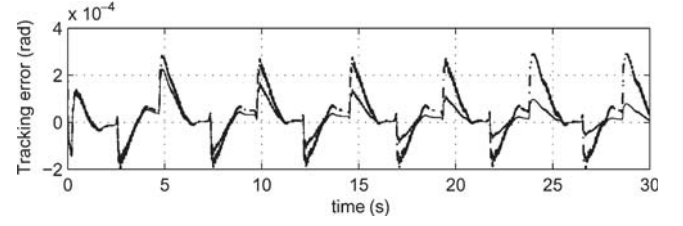


Fig. 4. Tracking performance (solid line: proposed method, dashed-dotted line: deterministic robust controller).

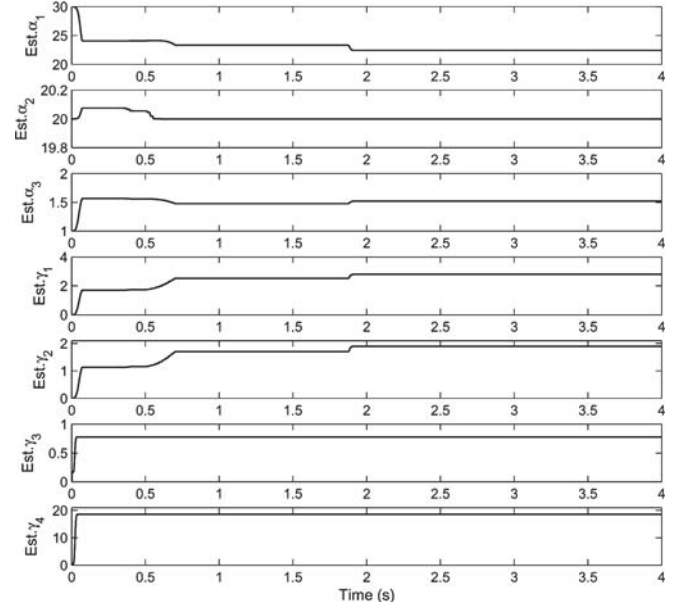


Fig. 5. Parameter estimates.

determined the value of T_{\max} , the pitch-axis is forced to run at a constant velocity. Then from (4), one has $\sigma_0 z = g(x_2) \text{sgn}(x_2)$. Consequently, the value of T_{\max} is estimated as $T_{\max} = 0.2 \text{ Nm}$.

From the definition of control robust term u_{s2} , one knows that the control gain is determined by the bounds of unknown parameters. It indicates that if the bounds are chosen too loosely, the final controller may be very conservative. Even more, in the real application, it will lead the controller to become saturated and make the system unstable. As a result, we first estimate the unknown parameters roughly in the system through offline identification. Then the lower bound and upper bound are set as less or more than 10% of the estimates, respectively [28], [29]. Actually, in real application, the feedback gain of the robust controller term is easily determined by try-and-error method.

C. Experimental Results

In our experiment, the unbalance torque caused by gravitational torque is considered as the main external disturbance. The constant offset is estimated online. Thus, the value of δ is supposed as $\delta \leq 0.05$. The controller parameters are chosen as follows:

$$k_p = 5, k_{s1} = 5, u_{s2} = -k_{s2}e_\varepsilon, k_{s2} = 10 \\ \Gamma = \text{diag}(800, 20, 50, 50, 10, 10, 1e4), \varepsilon = 0.001, b = 2.$$

Noting the definition of u_{s2} , one has $k_{s2} = h/(2\alpha_1 \min \varepsilon_1)$. In the real application, the value of k_{s2} is chosen by try-and-error method. The desired trajectory is shown in Fig. 3 (i.e., with maximum velocity ± 4 rad/s and maximum acceleration of ± 10 rad/s²). The tracking performance and control effort are shown in Fig. 4 and Fig. 5. (The curves of parameter estimates and error e_2 are omitted due to the limited space.)

From Fig. 4, we can see that the tracking error decreases gradually because of the parameter adaptation. As seen from the Fig. 5, the control effort varying between $+5$ V and -5 V is up to half of the limits. This indicates that a more accurate tracking error could be obtained if the control gain increases.

VI. CONCLUSION

We have developed an adaptive robust control (ARC) scheme with dynamic friction compensation for a DC servomechanism. This scheme utilizes a Lipschitzian property to re-parameterize the plant, and thus the number of parameters to be updated in the ARC is equal to the number of unknown parameters of the plant. Moreover, the friction state observer is not needed in this method. Thus, the resulting control algorithm is convenient for engineering application. It has been proved that the proposed method can not only guarantee desired transient performance for the system, but also make the magnitude of steady-state tracking error arbitrarily small in the presence of parametric uncertainties only. The experimental results show that the proposed ARC scheme can achieve desirable control performance for the servo mechanisms with dynamic friction effects.

APPENDIX A

Proof: Let $\beta' \in \Omega_\beta$ and $\beta'' \in \Omega_\beta$ be fixed. Define $\eta(\rho) = (1 - \rho)\beta' + \rho\beta''$ for all $\rho \in [0, 1]$. It is easy to verify that $\eta(\rho) \in \Omega_\beta$. Since, in the domain $R \times \Omega_\beta$, the function $f(x_2, \beta)$ is continuously differentiable in β for any fixed $x_2 \in R$, we can conclude that $f(x_2, \eta(\rho))$ is also continuously differentiable in ρ . According to the mean value theorem, there exists $\rho_1 \in [0, 1]$ such that

$$\begin{aligned} f(x_2, \beta'') - f(x_2, \beta') &= \frac{\partial f(x_2, \beta)}{\partial \beta} \frac{\partial \beta}{\partial \rho} \Big|_{\rho=\rho_1} \\ &= \frac{\partial f}{\partial \beta}(x_2, \eta(\rho_1)) (\beta'' - \beta'). \end{aligned}$$

Since $\eta(\rho_1) \in \Omega_\beta$, we have

$$\begin{aligned} |f(x_2, \beta') - f(x_2, \beta'')| &\leq \sum_{i=1}^4 |\beta'_i - \beta''_i| \max_{0 < \rho < 1} \left| \frac{\partial f}{\partial \beta_i}(x_2, \eta(\rho)) \right| \\ &\leq \sum_{i=1}^4 |\beta'_i - \beta''_i| \max_{\beta \in \Omega_\beta} \left| \frac{\partial f}{\partial \beta_i}(x_2, \beta) \right|. \end{aligned}$$

From Definition 1 and the above inequality, we can obtain the result of Lemma 2.

APPENDIX B

Proof: The proof of Lemma 4 is divided into two parts, i.e., i) $e_\varepsilon \in R \setminus L_{V_\varepsilon}(\varepsilon')$; ii) $e_\varepsilon \in L_{V_\varepsilon}(\varepsilon')$.

i) Consider the case of $e_\varepsilon \in R \setminus L_{V_\varepsilon}(\varepsilon')$. According to (34), the states outside $L_{V_\varepsilon}(\varepsilon')$ satisfy $V_\varepsilon(e_\varepsilon) > \varepsilon'$, which implies $|e_\varepsilon| > (\sqrt{3} - 1)\varepsilon/\sqrt{3}$. From the definition of e_ε , it can be derived that $|e_2| > \varepsilon$. Then, according to (26), we obtain that $\partial e_\varepsilon / \partial e_2 = 1$. Substituting (31) into (30), we have

$$\begin{aligned} \dot{V}_\varepsilon &\leq e_\varepsilon \frac{\partial e_\varepsilon}{\partial e_2} [-k_{s1}e_\varepsilon + \alpha_1(u_a + u_{s2}) - \alpha_2x_2 - \alpha_3 \\ &\quad + \sigma_\varepsilon(e_2)T_{\max}(L^T(x_2)\gamma + f(x_2, \beta_{\min})) \\ &\quad - \dot{x}_{2eq} + \Delta] \\ &= -k_{s1}e_\varepsilon^2 + e_\varepsilon [\alpha_1u_{s2} - \tilde{\alpha}_1u_a + \tilde{\alpha}_2x_2 + \tilde{\alpha}_3 \\ &\quad - \sigma_\varepsilon(e_2)T_{\max}L^T(x_2)\tilde{\gamma} + \Delta] \\ &\quad \forall e_\varepsilon \in R \setminus L_{V_\varepsilon}(\varepsilon'). \end{aligned} \quad (36)$$

Combining with (33) and (35), it follows that

$$\dot{V}_\varepsilon \leq -\lambda V_\varepsilon + \varepsilon_1 < -\lambda \varepsilon' + \varepsilon_1 \leq -\lambda v_0 \quad \forall e_\varepsilon \in R \setminus L_{V_\varepsilon}(\varepsilon'). \quad (37)$$

Thus, it can be concluded that the trajectories starting outside $L_{V_\varepsilon}(\varepsilon')$ will enter $L_{V_\varepsilon}(\varepsilon')$ in a finite time.

ii) Consider the case of $e_\varepsilon \in L_{V_\varepsilon}(\varepsilon')$. In this case, we have

$$\begin{aligned} \dot{V}_\varepsilon &\leq e_\varepsilon \frac{\partial e_\varepsilon}{\partial e_2} [-k_{s1}e_\varepsilon + \alpha_1(u_a + u_{s2}) - \alpha_2x_2 - \alpha_3 \\ &\quad + \sigma_\varepsilon(e_2)T_{\max}(L^T(x_2)\gamma + f(x_2, \beta_{\min})) \\ &\quad - \dot{x}_{2eq} + \Delta] \\ &\leq \frac{\partial e_\varepsilon}{\partial e_2} (-\lambda V_\varepsilon + \varepsilon_1) \quad \forall e_\varepsilon \in L_{V_\varepsilon}(\varepsilon'). \end{aligned} \quad (38)$$

From (26), one has $\partial e_\varepsilon / \partial e_2 \geq 0$. Then, noting that $\varepsilon' > \varepsilon_1/\lambda$, in view of (38), it follows that

$$\dot{V}_\varepsilon \leq 0 \quad \forall e_\varepsilon \in L_{V_\varepsilon}(\varepsilon') \setminus L_{V_\varepsilon}(\varepsilon_1/\lambda) \quad (39)$$

which implies that when $e_\varepsilon \in L_{V_\varepsilon}(\varepsilon') \setminus L_{V_\varepsilon}(\varepsilon_1/\lambda)$, the Lyapunov function V_ε must be decreasing. Thus, the states inside $L_{V_\varepsilon}(\varepsilon')$ will not escape $L_{V_\varepsilon}(\varepsilon')$ and remain in it thereafter.

Combining the analysis in i) and ii), we can obtain the results in Lemma 4.

APPENDIX C

Proof: From Lemma 4, we know that, no matter what the initial conditions are, e_ε will enter the level set $L_{V_\varepsilon}(\varepsilon')$ after a finite time of running. It implies that $|e_\varepsilon| \leq \sqrt{2\varepsilon'}$, $\forall t \geq t_0$, $\exists t_0 > 0$. From the definition of e_ε , we have $e_2 = e_\varepsilon + c(e_2)$. Since $|c(e_2)| \leq \varepsilon$, it can be concluded that $|e_2(t)| \leq \sqrt{2\varepsilon'} + \varepsilon$, $\forall t \geq t_0$, $\exists t_0 > 0$, which completes the proof of i).

Then, we prove ii). Define a positive definite function V_θ as

$$V_\theta = \frac{1}{2}e_\varepsilon^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}. \quad (40)$$

If $\Delta = 0$, then from (6), (20), (31) and (34), the derivative of V_θ can be derived as

$$\begin{aligned}\dot{V}_\theta &\leq \frac{\partial e_\varepsilon}{\partial e_2} e_\varepsilon (-k_{s1} e_\varepsilon - \varphi^T \tilde{\theta} + \alpha_1 u_{s2}) \\ &\quad + \tilde{\theta}^T \Gamma^{-1} \text{Proj}_{\tilde{\theta}} \left(\Gamma \frac{\partial e_\varepsilon}{\partial e_2} e_\varepsilon \varphi \right) \\ &= -\frac{\partial e_\varepsilon}{\partial e_2} k_{s1} e_\varepsilon^2 + \frac{\partial e_\varepsilon}{\partial e_2} \alpha_1 u_{s2} e_\varepsilon \\ &\quad + \tilde{\theta}^T \Gamma^{-1} \left[\text{Proj}_{\tilde{\theta}} \left(\Gamma \frac{\partial e_\varepsilon}{\partial e_2} e_\varepsilon \varphi \right) - \frac{\partial e_\varepsilon}{\partial e_2} e_\varepsilon \varphi \right].\end{aligned}$$

Noting that $(\partial e_\varepsilon / \partial e_2) \geq 0$ and $\alpha_1 \geq a_{1\min} > 0$, According to the properties of projection operation and (32), the above inequalities can be simplified as

$$\dot{V}_\theta \leq -\frac{\partial e_\varepsilon}{\partial e_2} k_{s1} e_\varepsilon^2. \quad (41)$$

Define a quantity f_ε as $f_\varepsilon := k_{s1} e_\varepsilon^2 \partial e_\varepsilon / \partial e_2$. Then, integrating two sides of (41) from $t = 0$ to $t = \infty$ yields

$$\lim_{t \rightarrow \infty} \int_0^t f_\varepsilon(\tau) d\tau \leq V_\theta(0) - V_\theta(\infty). \quad (42)$$

Noting that $(1/2)\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$ is always bounded due to the projection mapping, from the result of Lemma 4, we know that $V_\theta(\infty)$ is bounded. Accordingly, it can be concluded that $\lim_{t \rightarrow \infty} \int_0^t f_\varepsilon(\tau) d\tau$ exists and is finite.

On the other hand, we can also derive that f_ε is uniformly continuous. The deduction is as follows. The definition of f_ε shows that $f_\varepsilon = 2k_{s1}(\partial e_\varepsilon / \partial e_2)V_\varepsilon$. Noting that V_ε is bounded, which can be justified by Lemma 4, we denote the upper bound of V_ε as B_V , i.e., $V_\varepsilon \leq B_V$. Besides, the (26) shows that $|\partial e_\varepsilon / \partial e_2| \leq 1$. From (36), (38) and Lemma 4, we know that \dot{V}_ε is bounded, thus V_ε is uniformly continuous, namely, for any $t_1, t \geq 0$ and $R_1 > 0$, there exists $\eta_1(R_1) > 0$ such that $|t - t_1| < \eta_1$ implies $|V_\varepsilon(t) - V_\varepsilon(t_1)| < R_1$. On the other hand, from the definition of $\partial e_\varepsilon / \partial e_2$, we can observe that $\partial e_\varepsilon / \partial e_2$ is also uniformly continuous, and thus for any $R_2 > 0$, there exist $\eta_2(R_2) > 0$ such that $|t - t_1| < \eta_2$ implies $|(\partial e_\varepsilon / \partial e_2)(t) - (\partial e_\varepsilon / \partial e_2)(t_1)| < R_2$. For any $R > 0$, we select R_1 and R_2 such that $2k_{s1}(R_1 + B_V R_2) \leq R$. Then, it can be obtained that, if $|t - t_1| < \eta := \min(\eta_1, \eta_2)$, then

$$\begin{aligned}|f_\varepsilon(t_1) - f_\varepsilon(t)| &= 2k_{s1} \left| \frac{\partial e_\varepsilon}{\partial e_2}(t_1)V_\varepsilon(t_1) - \frac{\partial e_\varepsilon}{\partial e_2}(t)V_\varepsilon(t) \right| \\ &\leq 2k_{s1} \left(|V_\varepsilon(t_1)| \left| \frac{\partial e_\varepsilon}{\partial e_2}(t_1) - \frac{\partial e_\varepsilon}{\partial e_2}(t) \right| \right. \\ &\quad \left. + \left| \frac{\partial e_\varepsilon}{\partial e_2}(t) \right| |V_\varepsilon(t_1) - V_\varepsilon(t)| \right) \\ &< 2k_{s1}(R_1 + B_V R_2) \leq R.\end{aligned}$$

Since R is arbitrarily chosen, f_ε is uniformly continuous.

Therefore, it can be concluded by the Barbatlat's lemma that $f_\varepsilon \rightarrow 0$ as $t \rightarrow \infty$. From (26) and the definition of e_ε , one has $\partial e_\varepsilon / \partial e_2 = 0$ if and only if $e_\varepsilon = 0$, and then we obtain $f_\varepsilon \rightarrow 0$ equals to $e_\varepsilon \rightarrow 0$. In view of the definition of e_ε , it follows that

$|e_2(t)| \leq (\sqrt{3} - 1)\varepsilon/2$ as $t \rightarrow \infty$, which completes the proof of result ii).

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