

# Transactions of the Institute of Measurement and Control

<http://tim.sagepub.com/>

---

## **Flocking control for multi-agent systems with stream-based obstacle avoidance**

Qiang Wang, Jie Chen, Hao Fang and Qian Ma

*Transactions of the Institute of Measurement and Control* published online 26 September 2013

DOI: 10.1177/0142331213503864

The online version of this article can be found at:

<http://tim.sagepub.com/content/early/2013/09/26/0142331213503864>

---

Published by:



<http://www.sagepublications.com>

On behalf of:



[The Institute of Measurement and Control](#)

Additional services and information for *Transactions of the Institute of Measurement and Control* can be found at:

Email Alerts: <http://tim.sagepub.com/cgi/alerts>

Subscriptions: <http://tim.sagepub.com/subscriptions>

Reprints: <http://www.sagepub.com/journalsReprints.nav>

Permissions: <http://www.sagepub.com/journalsPermissions.nav>

>> [OnlineFirst Version of Record](#) - Sep 26, 2013

[What is This?](#)



# Flocking control for multi-agent systems with stream-based obstacle avoidance

Transactions of the Institute of  
Measurement and Control  
0(0) 1–8

© The Author(s) 2013

Reprints and permissions:

sagepub.co.uk/journalsPermissions.nav

DOI: 10.1177/0142331213503864

tim.sagepub.com



Qiang Wang<sup>1,2</sup>, Jie Chen<sup>1,2</sup>, Hao Fang<sup>1,2</sup> and Qian Ma<sup>3</sup>

## Abstract

The problems of flocking with both connectivity maintenance and obstacle avoidance for the network of dynamic agents are addressed. In the case where the initial network is connected, a decentralized flocking control protocol is proposed to enable the group to asymptotically achieve the desired stable flocking motion using artificial potential functions combined with stream functions, which could not only maintain the network connectivity of the dynamic multi-agent systems for all time but also make all the agents avoid obstacles smoothly without trapping into local minima. Finally, nontrivial simulations and experiments are worked out to verify the effectiveness of the theoretical methods.

## Keywords

Flocking, connectivity maintenance, obstacle avoidance, stream function

## Introduction

In the past few years, coordinated flocking has emerged as a robust way for addressing a wide variety of spatially distributed tasks ranging from extraterrestrial exploration, surveillance, rescue operations and military missions to cooperative construction, and so on. Considerable efforts have been made in analysis and modeling of the collective dynamics for a better understanding of how a group of mobile agents can perform complex tasks without centralized control, and how to design suitable distributed strategies to perform a collective task.

Flocking is characterized by decentralized control, local interaction and self-organization. It is called stable flocking when all the agents asymptotically approach the same velocity and maintain desired configuration formations, while collisions between agents and with obstacles are avoided, when moving towards a destination point with only limited local information. During the last decade, a large amount of literature has paid their attention to the algorithms and theories of two kinds of flocking control problem: free flocking and constrained flocking. The classical flocking model consisting of three heuristic rules of separation, cohesion and alignment has been proposed (Reynolds, 1987). A group of autonomous agents moving in the plane is considered and a class of local control laws is introduced which combined the artificial potential field with the velocity consensus to obtain stable flocking motion in both fixed and switching networks (Tanner et al., 2003; Tanner et al., 2007). Moreover, many other constraints have been added to the basic flocking algorithms for achieving various control objectives such as obstacle avoidance (Chang et al., 2003; Olfati-Saber, 2006; Fahimi et al., 2009) and connectivity maintenance (Zavlanos and Pappas, 2007; Zavlanos et al., 2009; Dimarogonas and

Johansson, 2010; Su et al., 2010; Wang and Wang, 2010). The problem of obstacle avoidance using gyroscopic forces for multi-agent systems is discussed, which relies on a centralized construction of the potential function in Chang et al. (2003). A complement to the traditional panel method is introduced to generate a more effective harmonic potential field for obstacle avoidance in dynamically changing environments, and a group of mobile robots working in an environment containing stationary and moving obstacles is considered (Fahimi et al., 2009). A comprehensive theoretical framework of distributed flocking control and a unified analytical look at Reynolds's rules are proposed (Olfati-Saber, 2006). Two cases of flocking in free-space and the presence of multiple obstacles are considered. Three flocking algorithms are presented: two for free flocking and one for constrained flocking, but these failed to maintain the connectivity of the interaction. However, most previous works make a basic assumption that there is a connected frequently switching topology during the evolution, which is difficult to implement in the engineering applications.

As a common inherent condition, it is often required that there is connectedness of the underlying communication network in these distributed strategies in which the information

<sup>1</sup>School of Automation, Beijing Institute of Technology, Beijing, PR China

<sup>2</sup>Key Laboratory of Complex System Intelligent Control and Decision, Beijing, PR China

<sup>3</sup>School of Automation, Nanjing University of Science and Technology, Nanjing, PR China

## Corresponding author:

Qian Ma, School of Automation, Nanjing University of Science and Technology, Nanjing, 210094, PR China

Email: qianmashine@gmail.com



exchange and sharing can be realized between neighbors to ensure reliable and efficient motion coordination. Therefore, it is significant to design decentralized motion control laws that enable the multi-agent systems to achieve desired cooperative tasks while maintaining the network connectivity. The problem of preserving the connectivity property of the network is solved by translating the connectivity condition to a projected graph Laplacian and guaranteeing the positive definiteness of the eigenvalues (Zavlanos and Pappas, 2007). A distributed control law is designed for connectivity maintenance objective via the use of decentralized navigation functions, which are bounded potential fields (Dimarogonas and Johansson, 2010). Furthermore, potential function method as an efficient technique is used to preserve the existing communication links for double-integrator agents in the papers (Zavlanos et al., 2009; Su et al., 2010). A bounded distributed controller is proposed to steer a group of agents to synchronization while avoiding collision as well as preserving connectivity among agents for all time (Wang and Wang, 2010).

To the best of our knowledge, one common method used in current literatures for multi-agent systems to realize obstacle avoidance and connectivity maintenance is to drive the neighboring agents to follow the (sometimes negated) gradient of an artificial potential field (APF) which is dependent on relative distance from each other, and is constructed such that the resulting vector field is exterior directed on the boundaries of the configuration space. However, a major drawback of the potential field method is the presence of local minima deviating from the globally optimal goal, which may lead to unexpected failure of task implementation. To overcome the aforementioned drawbacks, a decentralized cooperative control algorithm is proposed for multi-agent systems which could deal with flocking, connectivity maintenance and obstacle avoidance simultaneously. A hybrid mechanism for potential flow is presented to realize the agent coordination by integrating the advantages of APFs with harmonic stream functions for obstacle avoidance. Under the premise of the initial connectivity of the network topology, an APF that has the feature of attraction forces is used to achieve connectivity preserving among agents. Furthermore, a stream function is designed and combined with the APF to generate smooth and obstacle-avoidance trajectories, which guide all the agents to the target without trapping into local minima.

The remainder of the paper is organized as follows. In Section 2, the problem is formally formulated and several preliminaries are introduced. In Section 3, a decentralized cooperative control law is proposed to solve the problem of flocking with connectivity maintenance and obstacle avoidance under arbitrary initially connected network. Simulations and experiments are shown in Section 4 to validate the theoretical results. Finally, conclusions are drawn and future directions are stated in Section 5.

## Problem statement and preliminaries

### Problem formulation

Consider a group of  $n$  agents moving in the two-dimensional Euclidean plane with double integrator dynamics. A continuous-time model of the system is described by

$$\begin{aligned}\dot{x}_i &= v_i \\ \dot{v}_i &= u_i\end{aligned}\quad (1)$$

where  $x_i \in \mathcal{R}^2$  is the position vector of agent  $i$ ,  $v_i \in \mathcal{R}^2$  is the velocity vector,  $u_i \in \mathcal{R}^2$  is the control input acting on agent  $i$ . The relative position vector between agents  $i$  and  $j$  is represented by  $x_{ij} = \|x_i - x_j\|$ . In order to fulfill the control objective,  $u_i \in \mathcal{R}^2$  should be designed to achieve the stable flocking motion, which not only makes all the agents asymptotically approach the same velocity and the desired inter-agent distance stabilization, but also realizes obstacle avoidance under the condition of preserving network connectivity at all times when the network is initially connected.

Suppose that all the agents have the same sensing radius  $R$ . The underlying switching network can be represented by a time-varying undirected graph  $G(t) = \{V, E(t)\}$  consisting of a vertex set  $V = \{1, 2, \dots, n\}$ , indexed by the group of agents and an edge set  $E(t) = \{(i, j) | \|x_{ij}(t)\| \leq R\}$ , containing unordered pairs of nodes that represents neighboring relations. The set  $N_i(t) = \{j | (i, j) \in E(t)\}$  is the set of agent  $i$ 's neighbors.

### Preliminaries

The following definitions and theorems are derived from Okiishi et al. (2006). Incompressible, inviscid, irrotational fluid flow is discussed to describe potential functions in which the flow is always along the gradient of the fluid potential. Only incompressible fluid is discussed in this paper.

**Definition 2.1 (stream function and streamline):** The stream function is defined for two-dimensional flows of various kinds. The stream function can be used to plot streamlines, which represent the trajectories of particles in a steady flow. Streamlines are perpendicular to equipotential lines.

Consider a two-dimensional incompressible flow; the continuity equations in Cartesian coordinates are given by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

where  $u, v$  are the components of the fluid velocity in two-dimensional  $x$ - $y$  plane. The partial differential equation still has two unknown function,  $u$  and  $v$ . However, if a new function  $\Psi$  is arbitrarily defined as,

$$u = -\frac{\partial \Psi}{\partial y}, v = \frac{\partial \Psi}{\partial x} \quad (3)$$

Then the continuity equation becomes

$$\frac{\partial}{\partial x} \frac{\partial \Psi}{\partial y} + \frac{\partial}{\partial x} \left( -\frac{\partial \Psi}{\partial y} \right) = \frac{\partial^2 \Psi}{\partial x \partial y} - \frac{\partial^2 \Psi}{\partial y \partial x} = 0 \quad (4)$$

This new function  $\Psi$  is defined as a stream function. Note that if we can find a stream function  $\Psi$  that meets the equation (3), then the continuity equation need not be solved.

The streamlines are lines where the stream function is constant. At every point in the flow field, streamlines are tangent to the velocity field. That is



$$\frac{dy}{dx} = \frac{v}{u} \quad (5)$$

What we have proved then is that the line  $\Psi = C$  ( $C$  indicates a constant value) is a streamline of the flow. Alternately the equation of a streamline is given by  $\Psi = C$ .

**Definition 2.2 (uniform flow):** The flow is defined as uniform flow when the velocity and other hydrodynamic parameters do not change from point to point at any instant of time in the flow field. Let the strength of flow  $U_\infty = C$ , the stream function for uniform flow can be easily calculated and is given by,

$$U_\infty = \frac{\partial \Psi}{\partial y}, v = \frac{\partial \Psi}{\partial x} = 0 \quad (6)$$

**Definition 2.3 (doublet flow)** A doublet flow is a superposition of a sink and a source with the same strength. The stream function of doublet flow is given by

$$\Psi = -K \frac{r^2 y}{(x^2 + y^2)} \quad (7)$$

where  $K$  is the strength of the doublet flow and  $r$  is the radius of the obstacle boundary.

**Remark 1.** The usefulness of the stream function lies in the fact that the velocity components in the  $x$ - and  $y$ -directions at a given point are given by the partial derivatives of the stream function at that point. Streamlines are curves that show the mean direction of the fluid at the same instant of time. The curves are tangential to the velocity vectors at any points occupying the streamline. They depict the motion of the different particles in the flow field at the same instant of time and show the direction a fluid element will travel in at any point in time. By definition, different streamlines at the same instant in a flow do not intersect, because a fluid particle cannot have two different velocities at the same point.

## Design of control laws

### Design of control laws using stream function

Since the connectivity of the network cannot be guaranteed as time evolves with only initial connectivity, corresponding potential functions should be designed to prevent initially interconnected agents from moving out of their communication range and avoid collisions. Furthermore, it is also required that all the agents reach a common velocity while maintaining the desired group configuration which is described by

$$\begin{cases} v_{ij}(t) = v_i(t) - v_j(t) \rightarrow 0 \\ x_{ij}(t) = x_i(t) - x_j(t) \rightarrow d_{ij} \end{cases} \forall i, j \in V \quad (8)$$

where  $d_{ij}$  is the desired distance between agents  $i$  and  $j$ . Hence, the potential function for stable flocking can be devised as follows:

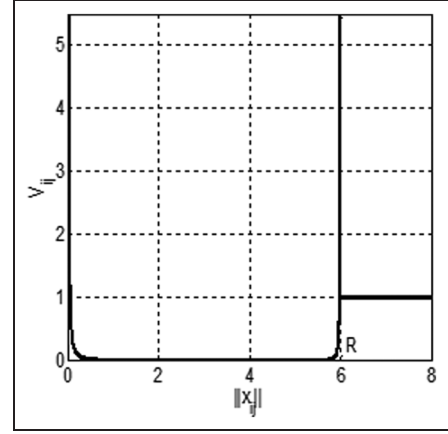


Figure 1. Stable flocking APF.

$$V_{ij}(\|x_{ij}\|) = \begin{cases} \left(\frac{1}{\|x_{ij}\|} - \frac{1}{d_{ij}}\right)^{c_1} \frac{1}{(R^2 - \|x_{ij}\|^2)^{c_2}} & 0 \leq \|x_{ij}\| \leq R \\ c & \|x_{ij}\| > R \end{cases} \quad (9)$$

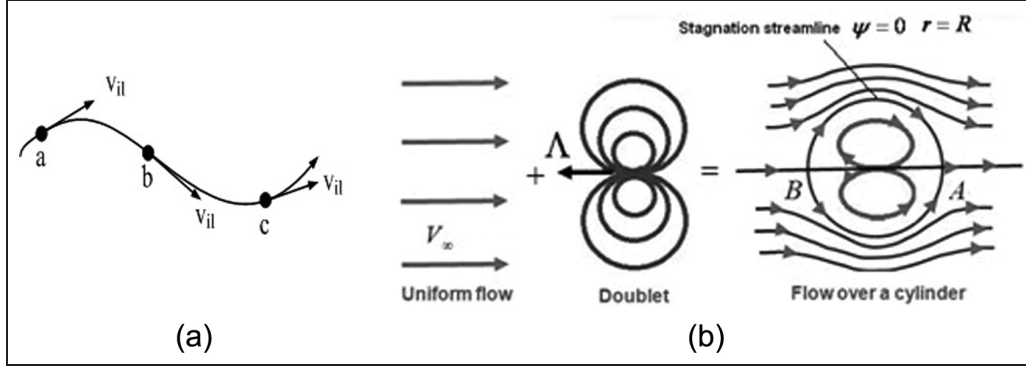
where  $c_1 \geq 2$ ,  $c_2 \geq 1$  and  $c \geq 0$ .  $V_{ij}(\|x_{ij}\|)$  is a nonnegative, piecewise continuous, differentiable for  $(0, R)$  and unbounded function of the distance  $\|x_{ij}\|$ , which satisfies  $V_{ij}(\|x_{ij}\|) \rightarrow \infty$ , as  $\|x_{ij}\| \rightarrow 0$  or  $\|x_{ij}\| \rightarrow R$ . The main difference between the potential function  $V_{ij}$  here from Tanner et al. (2007) is that  $V_{ij}$  tends to infinite when the distance between agents  $i$  and  $j$  tends to  $R$ , which can guarantee the preservation of all the initial links (see Figure 1).

Note that the potential function (9) is continuously differentiable in the interval  $(0, R)$  even if a new edge is added into  $E(t)$ . Hence the proposed APF enables the system to approach the stable flocking configuration and preserve the network connectivity simultaneously.

Moreover, as is well known, a fundamental drawback in the application of the APF method is the presence of local minima, which could force the agents to rest when approaching the obstacles and fail to achieve the desired task. In order to avoid obstacles, the concept of fluid mechanics is used and a virtual leader is introduced for each agent that is free of collision from obstacles. The basic idea is to regard the agents as part of the flow, and take the streamlines as the reference trajectories to be followed by the agents (see Figure 2(a)). According to the definition of the streamline, the velocity vectors of any points occupying the streamline can be obtained. Thus, the points on the streamline can be used as virtual leaders which lead each agent avoid obstacles along the smooth streamlines. For simplicity and without loss of generality, the cylinder-shaped obstacle centered at the origin with radius  $r$  is considered and placed in two-dimensional incompressible flow. Flow past a circular cylinder can be obtained by combining uniform flow with a doublet flow. Let the strength of the uniform flow and the doublet flow be  $U$  and  $K$ , respectively. The superimposed stream function is given according to definition 2.2 and definition 2.3

$$\Psi = \Psi_{\text{uniform flow}} + \Psi_{\text{doublet}} = Uy - K \frac{r^2 y}{(x^2 + y^2)} \quad (10)$$





**Figure 2.** Streamlines around cylinder-shaped obstacle.

Note that there  $x^2 + y^2 = r^2$  holds on the boundary of the obstacle, thus  $K = U$ . It is obvious that for the stream function on the surface of the cylinder  $\Psi = 0$ , since the streamline which passes through the stagnation point has a value of zero. The stream function for flow past a circular cylinder becomes

$$\Psi = Uy \left[ 1 - \frac{r^2}{(x^2 + y^2)^2} \right] \quad (11)$$

The plot of the streamlines around the obstacles is shown in Figure 2(b), which illustrates an example of the stream function of single static cylinder obstacle placed in two-dimensional incompressible flow, the lines represent the streamlines, and the circle AB is the boundary or radius of the obstacle. Note that the paths actually generated by following the streamlines tend to be smooth due to the tangent boundary condition. The point on the instantaneous streamlines can be used as virtual leaders and could guide the associated agents to avoid obstacles safely and smoothly.

When each agent  $i$  enters in the flow potential range of the obstacle, their orthogonal projection point on the streamlines will be selected as a set of the virtual leaders, the velocity of the virtual leaders is assigned the speed  $v_{il}$ , which is equal to the velocity of the stream field around the obstacle.

$$v_{il} = \nabla \Psi = u + iv = -\frac{\partial \Psi}{\partial y} + i \frac{\partial \Psi}{\partial x} \quad (12)$$

The potential field for consensus tracking between agents  $i$  and their virtual leader is denoted by  $V_{il}$  which is shown as below

$$V_{il} = \frac{1}{2} \left( \frac{1}{R - \|x_i - x_{il}\|} - \frac{1}{R} \right)^2 \quad (13)$$

The relative position vector between agents  $i$  and its virtual leader is represented by  $\hat{x}_i = x_i - x_{il}$

The explicit control input is chosen as below:

$$u_i = \left( - \sum_{j \in N_i(t)} \nabla_{\hat{x}_i} V_{ij}(\|\hat{x}_i - \hat{x}_j\|) - \sum_{j \in N_i(t)} a_{ij}(t)(\hat{v}_i - \hat{v}_j) \right) + (-\nabla_{\hat{x}_i} V_{il}(\|\hat{x}_i\|) - k_1 \hat{x}_i - k_2 \hat{v}_i + \dot{\hat{v}}_{il}) \quad (14)$$

where  $k_1, k_2 > 0$  are scalar control gains,  $N_i(t)$  is the time dependent neighborhood of agent  $i$  at time  $t$ , the relative velocity vectors between agents  $i, j$  and their virtual leaders are represented by  $\hat{v}_i = v_i - v_{il}$  and  $\hat{v}_j = v_j - v_{jl}$  respectively.  $A(t)$  is defined as follows:

$$A(t) = [a_{ij}(t)] = \begin{cases} 0 & \text{if } ((a_{ij}(t) = 0) \wedge (\|x_{ij}(t)\| \geq R - \delta)) \\ & \text{or } ((a_{ij}(t) = 1) \wedge (\|x_{ij}(t)\| \geq R)) \\ 1 & \text{otherwise} \end{cases} \quad (15)$$

where  $0 < \delta < R$  is a constant switching threshold.

**Remark 2.** Connectivity maintenance is not a necessary condition for multi-agent flocking control. If the system is jointly connected in any given time interval, then each agent will almost surely converge on the target. However, jointly connected interconnection topologies would make it more difficult to achieve the flocking process, and its convergence speed is significantly slower than that of connectivity maintenance. Thus, connectivity maintenance is more practical for multi-agent flocking control from the practical application and implementation aspects.

**Remark 3.** If only the connectivity maintenance is applied in the multi-agent system without using stream-based obstacle avoidance, then the obstacle avoidance cannot be achieved due to the presence of local minima. If only obstacle avoidance is considered in the system, the connectivity of the multi-agent network may be broken, and then the cooperative control task would fail eventually. Therefore, the combination of connectivity maintenance and obstacle avoidance has significant advantages over those previous results which only considered connectivity maintenance or obstacle avoidance in several practical applications.

### Stability analysis

**Theorem 3.1.** Consider a system of  $n$  agents with dynamics (1), each agent is steered by control law (14) and the neighboring graph is initially connected. Then the desired stable flocking motion can be achieved when all the agents asymptotically approach the same velocity and collisions between agents and with obstacles are avoided.



**Proof:** Consider the positive semi-definite function given as follows:

$$J = \frac{1}{2} \sum_{i=1}^n \left( \sum_{j \in N_i(t)} V_{ij}(\|\hat{x}_i - \hat{x}_j\|) \right) + \sum_{i=1}^n (V_{il}(\|\hat{x}_i\|)) \quad (16)$$

$$+ \frac{1}{2} \sum_{i=1}^n (\hat{v}_i^T \hat{v}_i) + \frac{1}{2} k_1 \sum_{i=1}^n (\hat{x}_i^T \hat{x}_i)$$

where  $V_{il}$  represents the potential field between agent  $i$  and its virtual leader.

Taking the time derivative of  $J$ , we have

$$\dot{J} = \sum_{i=1}^n \hat{v}_i^T \left( \sum_{j \in N_i(t)} \nabla_{\hat{x}_i} V_{ij}(\|\hat{x}_i - \hat{x}_j\|) \right) + \sum_{i=1}^n \hat{v}_i^T \nabla_{\hat{x}_i} V_{il}(\|\hat{x}_i\|) \quad (17)$$

$$+ \sum_{i=1}^n \hat{v}_i^T (\dot{v}_i - \dot{v}_{il}) + k_1 \sum_{i=1}^n \hat{v}_i^T \hat{x}_i$$

$\dot{v}_i = u_i$  is the control input of agent  $i$  given in (14), and therefore

$$\dot{J} = \sum_{i=1}^n \hat{v}_i^T \left( \sum_{j \in N_i(t)} \nabla_{\hat{x}_i} V_{ij}(\|\hat{x}_i - \hat{x}_j\|) \right) \quad (18)$$

$$+ \sum_{i=1}^n \hat{v}_i^T \nabla_{\hat{x}_i} V_{il}(\|\hat{x}_i\|) + \sum_{i=1}^n \hat{v}_i^T (u_i - \dot{v}_{il}) + k_1 \sum_{i=1}^n \hat{v}_i^T \hat{x}_i$$

$$= \sum_{i=1}^n \left( \hat{v}_i^T \left( - \sum_{j \in N_i(t)} a_{ij}(t)(\hat{v}_i - \hat{v}_j) - k_2 \hat{v}_i \right) \right)$$

$$\dot{J} = -k_2 \sum_{i=1}^n \hat{v}_i^T \hat{v}_i - \sum_{i=1}^n \left( \hat{v}_i^T \sum_{j \in N_i(t)} a_{ij}(t)(\hat{v}_i - \hat{v}_j) \right)$$

$$= -\hat{v}^T ((k_2 I_n + L_n(t)) \otimes I_2) \hat{v} \leq 0$$

where  $L_n(t)$  is the graph Laplacian associated with the undirected graph  $G(t)$ , and  $\otimes$  denotes the Kronecker production  $\hat{v} = [\hat{v}_1^T, \hat{v}_2^T, \dots, \hat{v}_n^T]^T$ . Thus,  $\dot{J} \leq 0$ , and  $\dot{J} = 0$  if and only if  $\hat{v}_i = 0$  for each  $i \in N$ . Specifically,  $\dot{J} = 0$  implies that  $\hat{v}_1 = \hat{v}_2 = \dots = \hat{v}_n = 0$ . Therefore, the velocity of agent  $i$  and its virtual leader become the same, it follows that,  $v_i = v_{il}, \forall i$ . Moreover, since the potential function  $V_{ij}$  and  $V_{il}$  are unbounded at  $\|\hat{x}_{ij}\| = R$ , the connectivity between the agents and the connectivity between the agents and their virtual leaders could be maintained simultaneously. Furthermore, the virtual leaders are on the streamline around obstacles. Hence, for flow past a circular cylinder, all the agents will asymptotically achieve the same velocity; almost every final configuration except for a local maximum or saddle point locally minimizes each agent's global potential. The stable flocking configuration is achieved under the control law (14).

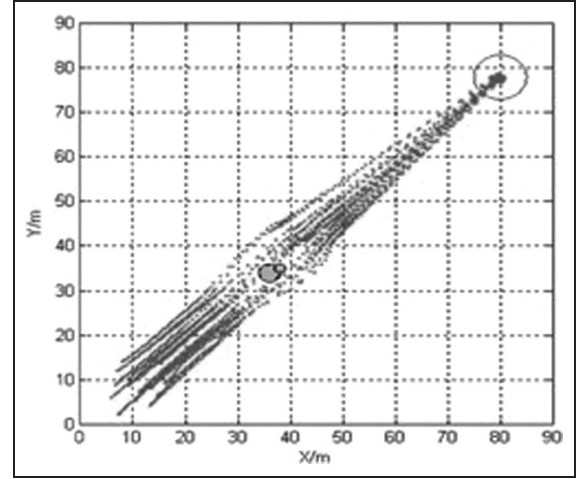
## Simulation and experiment

### Simulation

In this section, comparative numerical simulations are performed to illustrate the theoretical results obtained in previous sections and we compare our proposed obstacle avoidance flocking algorithm with the flocking algorithm proposed by

**Table 1.** Simulation parameters.

Parameter	Value
Size of sensing region	90 m × 90 m
Number of nodes	20
Simulation time	20 s
Target position	[80 m, 80 m]
Transmission range of nodes $R$	4.5 m
Desired distance $d_j$	3 m
Switching threshold $\delta$	0.7 m
Initial velocity of nodes	Randomly with arbitrary direction and magnitude within the range of (0, 5) m/s
Initial position of nodes	[0, 20] m × [0, 20] m within the circle of radius $R^* = 12$ m
The maximum velocity of node	0.5 m/s



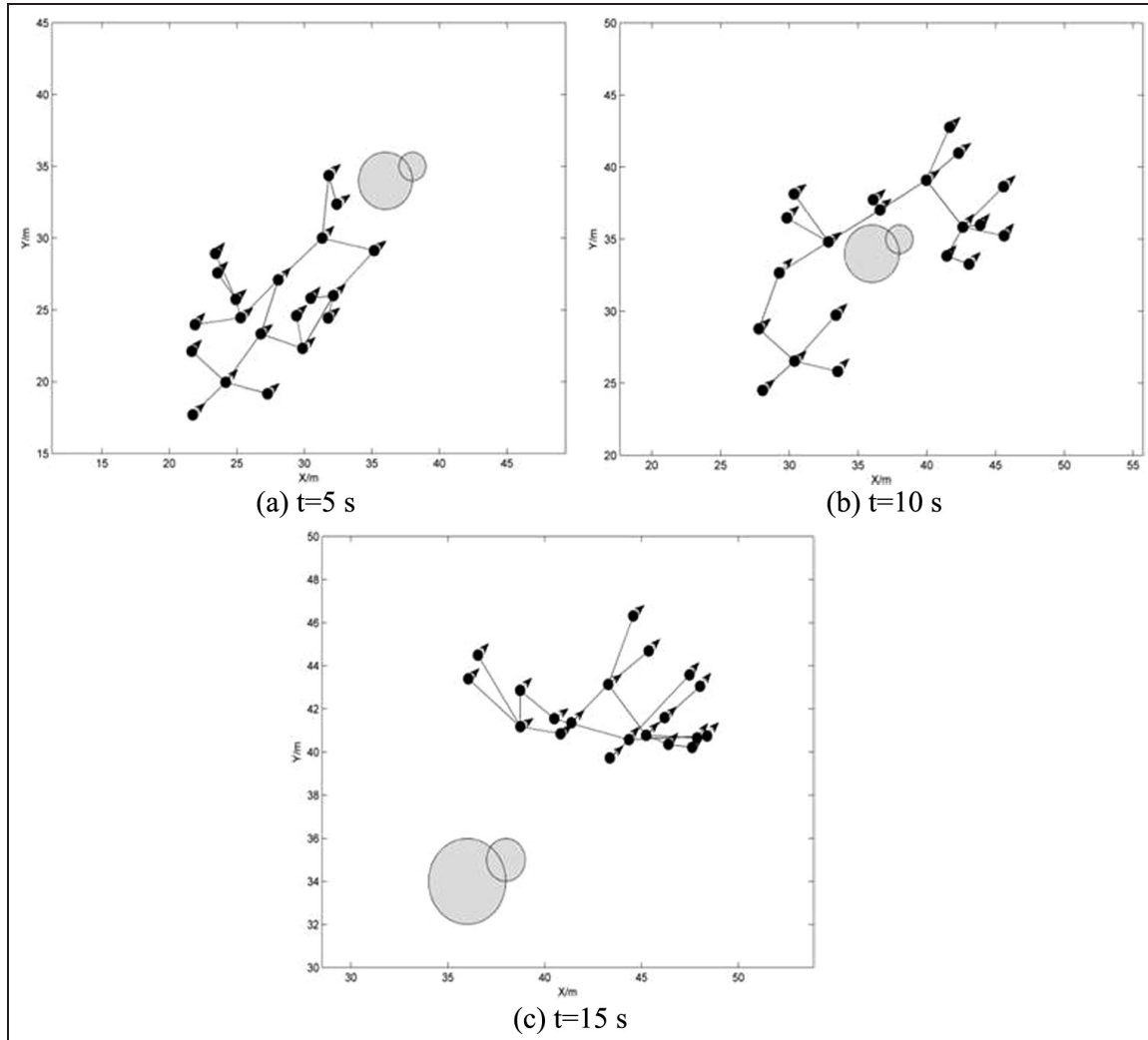
**Figure 3.** The entire trajectories of agents.

Olfati-Saber (2006). The following parameters applied in the simulation are shown in Table 1. The simulations are performed with dynamic model (1) in a two-dimensional plane, labeled with dots. The agents are arranged to construct the desired formation at the initial position, then the agents are controlled by the control law (14) to avoid the obstacles when they are in the process of stable flocking; the target points are reached eventually. In the simulations, the initial positions, velocities and links are randomly initialized to make the initial interactive network connected.

To validate the proposed methods, the simulation results of the flocking algorithms without connectivity maintenance (Olfati-Saber, 2006) and of that proposed in our work are all demonstrated in Figures 3–4. The locations of the obstacles, the agents and the target point are deliberately set in a straight line to simulate the circumstance where repulsive force of the obstacle and the attractive force of the target point may cause local minima or unforced crashes. The axes of the figure are appropriately chosen to illustrate the corresponding results.

Figure 3 describes the entire trajectory of the multi-agent system using our proposed algorithm. The far-end black circle



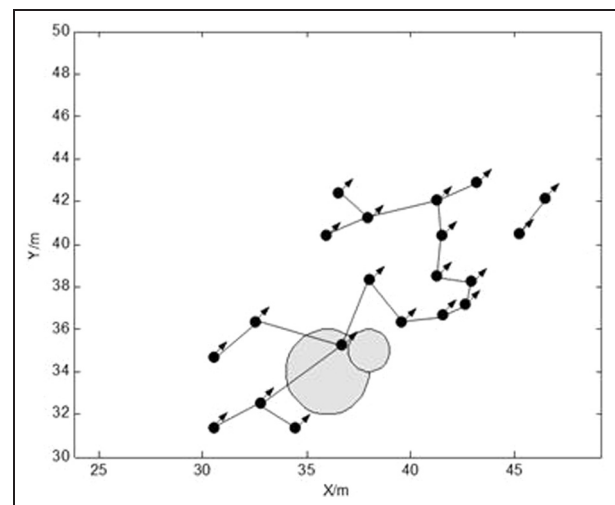


**Figure 4.** Simulation time sanpshots of cooperative flocking under control law (14).

in Figure 3 represents the actuating range of target position. It can be seen from the whole trajectory that the stable flocking motion is achieved. The connectivity of underlying time-varying interaction topology is preserved and the obstacle avoidance process is very smooth. Specifically, Figures 4 (a)–(c) show the state of twenty agents and neighboring relations between the agents, which are represented by solid lines at different instants using our proposed algorithm. Figure 5 shows the configuration of the group using the projection method for avoiding obstacle from Olfati-Saber (2006). As can be seen from Figure 5, one of the agents hit the obstacles, resulting in partition of the network and failing of obstacle avoidance for the multi-agent network, which is due to the problem of local minima.

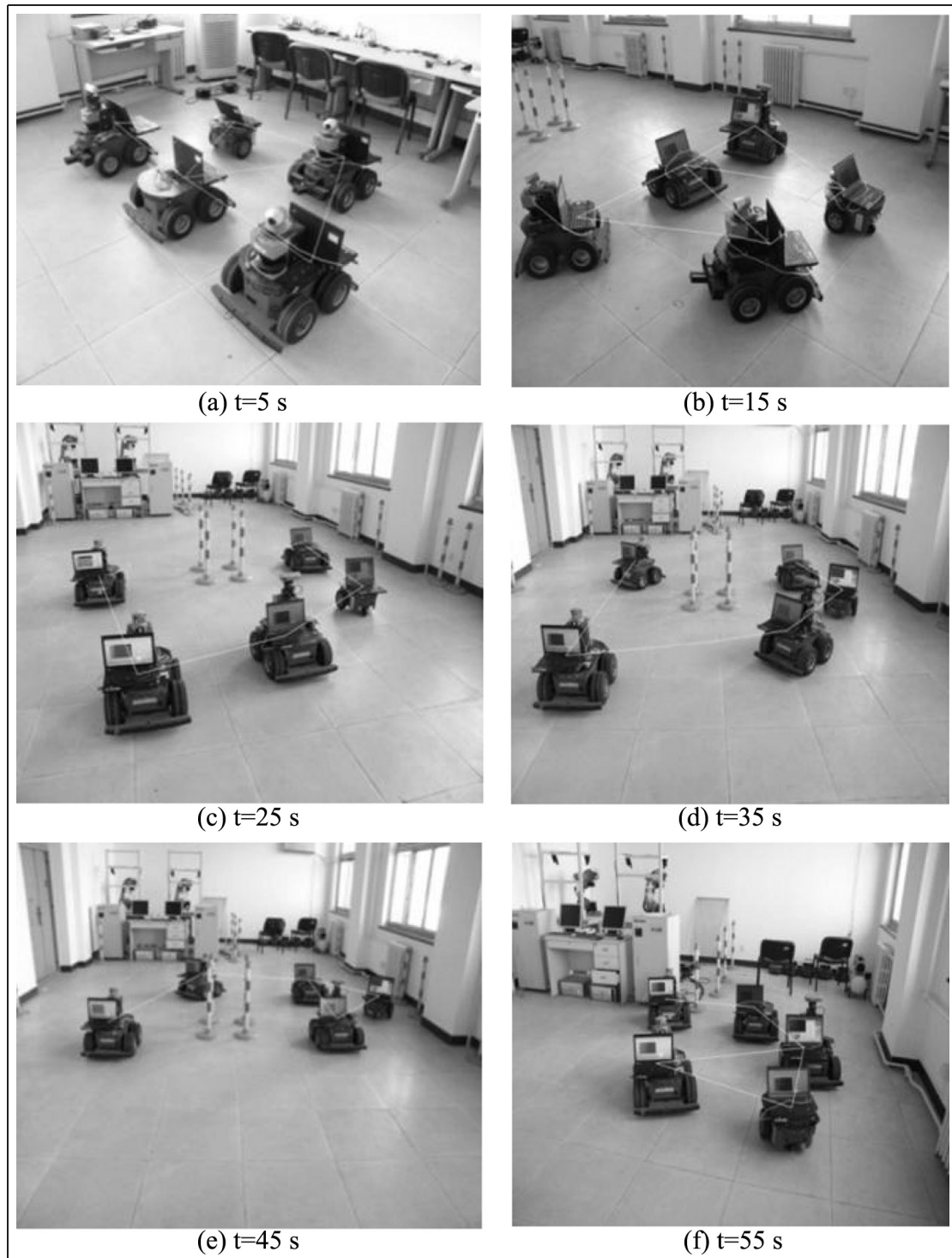
### Experiment

In this section, the experimental verification of flocking with real mobile robots is presented. The experiment of flocking control for multi-agent systems with connectivity



**Figure 5.** Simulation time snapshots of cooperative flocking at  $t=15$  s using control law of Olfati-Saber (2006).





**Figure 6.** Flocking of five mobile robots in indoor environment.

maintenance and obstacle avoidance based on the stream function and connectivity-preserving APF is carried out with four Pioneer3-AT mobile robots and one Pioneer3-DX mobile robot to validate the practical effectiveness of the proposed distributed flocking control algorithm. We assume that all the robots satisfy nonslipping and pure-rolling constraints

and each robot can obtain the information needed via its wireless communication equipment.

As is shown in Figure 6, the environment is a rectangular space with  $7\text{ m} \times 8\text{ m}$  in which four pillars are used as the obstacles. The target point locates at the right corner of the rectangular space. The process of experiment is shown in



Figures 6(a)–(f), which depict six typical snapshots of the flocking process within a time frame of 60 seconds. The initial positions and connections of the group are illustrated in Figure 6(a). Figure 6(b) shows the stable flocking process forced by the attraction/repulsion potentials with connectivity maintenance before avoiding obstacle. Figures 6(c)–(e) demonstrate the stable and smooth process of obstacle avoidance. Figure 6(f) shows the formation restoration of the system in steady state and moving towards the target point. It can be observed that despite the presence of nonholonomic dynamics, communication delays, noises, and so on, the desired flocking behavior is successfully achieved at last.

## Conclusion

In this paper, the flocking control problem for a network of dynamic agents with the purpose of connectivity maintenance and obstacle avoidance is investigated. A novel framework which combines the stream function with the artificial potential field is presented. The distinguishing feature of the proposed control law is that the stream function yields smooth trajectories for obstacle avoidance while the interactive potential guarantees the stability of the flocking motion. The control laws can not only make the agents achieve the velocity alignment and reach the desired configuration, but also fulfill the requirements of connectivity maintenance and obstacle avoidance. Future research will focus on the impact of communication link failure on the system as well as the flocking problem for multi-agent groups.

## Funding

This work was supported by National Science Fund for Distinguished Young Scholars (grant number 60925011), Projects of Major International (Regional) Joint Research Program NSFC (grant number 61120106010), the NSFC (grant number 61175112) and the Beijing Education Committee Cooperation Building Foundation Project.

## References

- Chang DE, Shadden SC, Marsden JE and Olfati-Saber R (2003) Collision avoidance for multiple agent systems. In: *Proceedings of 42nd IEEE conference on decision and control*, 2003, Vol. 1, pp. 539–543.
- Dimarogonas DV and Johansson KH (2010) Bounded control of network connectivity in multi-agent systems. *IET Control Theory and Applications* 4(8): 1330–1338.
- Fahimi F, Nataraj C and Ashrafiuon H (2009) Real-time obstacle avoidance for multiple mobile robots. *Robotica* 27(2): 189–198.
- Okiishi MY, Munson B and Young D (2006) *Fundamentals of Fluid Mechanics*. John Wiley & Sons.
- Olfati-Saber R (2006) Flocking for multi-agent dynamic systems: Algorithms and theory. *IEEE Transactions on Automatic Control* 51(3): 401–420.
- Reynolds CW (1987) Flocks, herds and schools: A distributed behavioral model. In: *ACM SIGGRAPH Computer Graphics* 21(4): 25–34.
- Su H, Wang X and Chen G (2010) Rendezvous of multiple mobile agents with preserved network connectivity. *Systems and Control Letters* 59(5): 313–322.
- Tanner HG, Jadbabaie A and Pappas GJ (2003a) Stable flocking of mobile agents, Part I: Fixed topology. In: *Proceedings of 42nd IEEE conference on decision and control*, 2003, Vol. 2, pp. 2010–2015.
- Tanner HG, Jadbabaie A and Pappas GJ (2003b) Stable flocking of mobile agents, Part II: dynamic topology. In: *Proceedings of 42nd IEEE conference on decision and control*, 2003, Vol. 2, pp. 2010–2015.
- Tanner HG, Jadbabaie A and Pappas GJ (2007) Flocking in fixed and switching networks. *IEEE Transactions on Automatic Control* 52(5): 863–868.
- Wang L and Wang X (2010) Flocking of mobile agents while preserving connectivity based on finite potential functions. In: *2010 8th IEEE International Conference on Control and Automation (ICCA)*, pp. 2056–2061.
- Zavlanos MM and Pappas GJ (2007) Potential fields for maintaining connectivity of mobile networks. *IEEE Transactions on Robotics* 23(4): 812–816.
- Zavlanos MM, Tanner HG, Jadbabaie A and Pappas GJ (2009) Hybrid control for connectivity preserving flocking. *IEEE Transactions on Automatic Control* 54(12): 2869–2875.