Mean square exponential stabilization of sampled-data Markovian jump systems

Guoliang Chen1,2 | Jian Sun1,2 | Jie Chen1,2

1School of Automation, Beijing Institute of Technology, Beijing, China
2Key Laboratory of Intelligent Control and Decision of Complex System, Beijing Institute of Technology, Beijing, China

Summary
In this paper, the problem of mean square exponential stabilization for sampled-data Markovin jump systems is studied. A time-scheduled Lyapunov functional consisting of a exponential-type looped function is constructed using segmentation technology and linear interpolation. Based on this new Lyapunov functional, a less conservative mean square exponential stability criterion is obtained such that a bigger maximum decay rate can be easily calculated. Meanwhile, the quantitative relationship among some system parameters, maximum sampling period, and decay rate is established. Moreover, a time-dependent state feedback sample-data controller is designed. Significant improvements of the proposed exponential-type time-scheduled Lyapunov functional method over some existing ones are verified by numerical examples.

KEYWORDS
Markovian jump system, mean square exponential stabilization, sampled-data control, time-scheduled Lyapunov functional

1 | INTRODUCTION

Markovian jump systems (MJSs) are a special class of stochastic and hybrid systems with the switching signals governed by a Markovian chain taking values in a finite set. The MJSs play an important role in describing many real world applications, such as economic systems,1 flight systems,2 power systems,3 communication systems,4 and networked control systems.5,6 Many important results have been presented for MJSs over the past decades, eg, dynamic output feedback controller design,7 robust stabilization,8,9 mean square stability analysis,10-13 sliding-mode controller design,14-16 and filtering design.17-22 From a practical implementation point of view, more and more attention has been paid to sampled-data systems based on the development of the digital control technology.23 Moreover, the last decade has witnessed an enormous interest in the study of networked and embedded control systems.24-31 With the emergence of event-based,32 self-triggered control techniques,33 and some existing problems (sampling jitters, packet dropouts, and so on) in networked control systems, the study of aperiodic sampled-data systems constitutes nowadays a very popular research topic in control area. Many new techniques have been developed, eg, input delay method,34,35 looped-functionals method,36 discrete-time system approach,37 integral quadratic constraint approach,38 and impulsive-switched-system approach.39
It is worthwhile noting that most of the results are based on deterministic systems while few of them for SDMJSs. Based on a piecewise continuous Lyapunov function, the problem of robust control for uncertain sampled-data systems with random jumping parameters was considered in the work of Hu et al.\textsuperscript{40} By input delay method, SDMJSs were transformed into Markov jump systems with polytopic uncertainties and sawtooth delays, and the mean-square exponential sampled-data controller was designed based on the extended dissipative definition in the work of Shen et al.\textsuperscript{41} Based on the passivity definition, nonfragile sampled-data controllers and adaptive fault-tolerant sampled-data controllers for SDMJSs with aperiodic sampling were designed in the works of Wu et al\textsuperscript{42} and Sakthivel et al.\textsuperscript{43} In the works of Geromel and Gabriel,\textsuperscript{44,45} the mean square stability criteria and the $H_2$ and $H_\infty$ performance of SDMJSs were addressed and a necessary and sufficient condition was proposed in their aforementioned work.\textsuperscript{43} From the aforementioned literature, we can find very few studies on the exponential stabilization problem of SDMJSs. It should be pointed out that how to estimate decay rate of SDMJSs better is still an important issue and has not been fully investigated. In addition, there is a very interesting question worth considering, i.e., what is the quantitative relationship between the sampling period and decay rate? Unfortunately, there is no relevant literature considering this issue. Moreover, from a dynamic system point of view, a time-dependent sampling controller is clearly more adaptable to dynamic changes in the system. However, this type of controller did not appear in the literature mentioned previously. Therefore, the aforementioned three issues motivates this study.

In this paper, the problem of mean square exponential stabilization for SDMJSs is studied. The main contributions of this paper can be summarized in four aspects. Firstly, inspired by region-dividing technique and time-scheduled multiple Lyapunov functionals for switched systems,\textsuperscript{46-48} by dividing the lower bound of sampling period into $L$ segments and using a linear interpolation formula, a new exponential-type time-scheduled Lyapunov functional consisting of an exponential-type looped-functional is constructed. Secondly, a less conservative exponential stability criterion is derived. Lastly, a detailed proof is given to guarantee that the mean square exponentially stability of the continuous-time MJSs by the discrete-time sampled-data feedback control and the quantitative relationship among some system parameters, maximum sampling period, and decay rate are established. Fourthly, time-dependent state feedback sampled-data controller is designed for SDMJSs.

Notation 1. Throughout this paper, $I$ denotes the identity matrix with appropriate dimension; $M^T$ represents the transpose of the matrix $M$; and $X > 0$ ($\geq 0$) means that $X$ is a symmetric positive-definite (positive-semidefinite) matrix. $\lambda_{\max}(X)$ and $\lambda_{\min}(X)$ denote the maximum and minimum eigenvalues, respectively. $\mathbb{R}$ denotes the set of real numbers; $\mathbb{R}^n$ denotes the set of $n$-dimensional real vector; $\mathbb{R}^+$ denotes the set of all nonnegative real numbers; and $\mathbb{N} = \{1, 2, \ldots\}$, $\mathbb{R}^{n \times n}$ is the set of $n \times n$ real symmetric matrices. $\mathbb{L}_2[0, \infty)$ stands for the space of square integrable functions on $[0, \infty)$. $\mathbb{E}()$ stands for the mathematical expectation. If $x \in \mathbb{R}^n$, then $|x|$ is its Euclidean norm. If $A$ is a matrix, $||A|| = \max\{|Ax| : |x| = 1\}$ be the operator norm; and $\text{He}\{A\} = A + A^T$, diag(...) denotes a block-diagonal matrix. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

\section{Problem Formulation and Preliminaries}

Consider an MJS

\begin{equation}
\begin{aligned}
\dot{x}(t) &= A(r(t))x(t) + B(r(t))u(t), \\
x(0) &= x_0, \quad r(0) = r_0,
\end{aligned}
\end{equation}

where $x(t) \in \mathbb{R}^n$ is the state vector of the system, $u(t) \in \mathbb{R}^m$ is the control vector of the system, and $r(t)$ is a continuous time Markovian process with right continuous trajectories and taking values in a finite set $I = \{1, 2, \ldots\}$ with transition probability matrix $\Pi \triangleq [\pi_{ij}]_{ij}$ given by

\begin{equation}
\begin{aligned}
\Pr\{r(t+h) = j| r(t) = i\} &= \begin{cases} 
\pi_{ij}h + o(h), & i \neq j \\
1 + \pi_{ii}h + o(h), & i = j,
\end{cases}
\end{aligned}
\end{equation}

where $h > 0$, $\lim_{h \to 0}(o(h)/h) = 0$. Here, $\pi_{ij} \geq 0$ is transition rate from mode $i$ at time $t$ to mode $j$ at time $t + h$ if $i \neq j$ and $\pi_{ii} = \sum_{j=1, j \neq i}^{s} \pi_{ij} \cdot A(r(t)) \in \mathbb{R}^{n \times n}$, $B(r(t)) \in \mathbb{R}^{n \times m}$ are system matrices. For convenience, we set $A(r(t)) = A_i$, $B(r(t)) = B_i$.

For system (1), the state of (1) is sampled when $t = t_k$, where $\{t_k\}$ represents a set of the sampling instants and satisfies

\begin{equation}
0 = t_0 < t_1 < t_2 < \ldots < t_k < \ldots, \quad t_k \in \mathbb{R}^+, \quad \forall k \in \mathbb{N}.
\end{equation}
The control input is given by
\[ u(t) = -K_0(t)x(t_k), \quad \forall t \in [t_k, t_{k+1}), \quad (4) \]
where \( K_0(t) \) is a time-dependent and Markovian jump mode-dependent state feedback sampled-data controller.

Substituting (4) into (1), we have a following sampled-data MJS (SDMJS)
\[
\begin{aligned}
\dot{x}(t) &= A_0x(t) - B_0K_0(t)x(t_k), \\
x(0) &= x_0, r(0) = r_0.
\end{aligned}
\]

The sampling period \( T_k \) is denoted by \( T_k = t_{k+1} - t_k \) and satisfies \( 0 < T_{\min} \leq T_k \leq T_{\max} \). The sampling does not change jump action of Markovian process.

**Definition 1.** MJS (1) is said to be mean square exponentially stable if
\[ \mathbb{E}\left\{ |x(t)|^2 \right\} \leq a e^{-\lambda t} |x(0)|^2 \]
for any finite \( x_0 \in \mathbb{R}^n \) and \( r_0 \in \mathcal{N} \), where \( x(t) \) is the trajectory of the system state from initial system state \( x_0 \) and initial mode \( r_0 \), and \( a, \lambda \) are positive constants. Here, \( \lambda \) is called the decay rate.

**Lemma 1.** (See the work of Gu et al.)
For a given positive definite matrix \( R \), for all continuous functions \( x \in [a, b] \to \mathbb{R}^n \), the following inequality holds:
\[ \int_a^b x^T(u)Rx(u)du \geq \frac{1}{b-a} \int_a^b x^T(u)du \int_a^b x(u)du. \]

**Lemma 2.** (See the work of Yan et al.) (Gronwall-Bellman inequality)
Let \( \theta(t) \) be a nonnegative function such that
\[ \theta(t) \geq a + b \int_0^t \theta(s)ds, 0 \leq t \leq T, \]
for some constants \( a, b \geq 0 \), then we have
\[ \theta(t) \geq ae^{bt}, 0 \leq t \leq T. \]

### 3 | STABILITY ANALYSIS FOR SDMJSS

In this section, for simplicity of vector and matrix representation, some reconstructed block matrices are denoted by
\[ e^T(t) = [x^T(t) \dot{x}^T(t) x^T(t_k)], \]
\[ e_\rho = [0_{nx(\rho-1)n} I_{nxn} 0_{nx(3-\rho)n}] \in \mathbb{R}^{nx3n}, \rho = 1, 2, 3. \]

For the sampling-data interval \([t_k, t_k + T_{\min})\), we divided the interval into \( L \) equal segments. The length of every segment is \( \delta = \frac{T_{\min}}{L} \). Setting \( \theta_0 = 0, \theta_q = q \delta, \lambda_{k,q} = [t_k + \theta_q, t_k + \theta_{q+1}), \lambda_{k,L} = [t_k + T_{\min}, t_{k+1}) \). There are \( \bigcup_{n=0}^{L-1} \lambda_{k,n} = [t_k, t_k + T_{\min}) \) and \( \lambda_{k,m} = \emptyset \) for any \( m \neq L \). Let \( \Pi(q) = P(t_k + \theta_q) \), since \( P(t) \) is piecewise linear in the interval \([t_k, t_k + T_{\min})\), and using a linear interpolation formula, for \( 0 \leq u \leq 1 \), we have the following Lyapunov functional.

Construct a Lyapunov functional candidate
\[ \mathcal{V}(t) = W(t) + v(t, t_{k+1}, x(t)), \]
where
\[
W(t) = \begin{cases}
V(x(t)), & t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1, \\
\Lambda(x(t)), & t \in \lambda_{k,L},
\end{cases}
\]
with
\[ V(x(t)) = e^{\delta t}x^T(t)P(t)x(t) \]
\[ \Lambda(x(t)) = e^{\delta t}x^T(t)P_Lx(t), \]
\[ \mathcal{V}(t) = \mathcal{V}(t_{k+1}) + \int_{t_k}^{t_{k+1}} \mathcal{W}(s)ds \]
\[ \mathcal{W}(s) = \begin{cases}
\dot{\mathcal{V}}(s), & t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1, \\
\Delta(\mathcal{V}(s)), & t \in \lambda_{k,L},
\end{cases}
\]
\[ \Delta(\mathcal{V}(s)) = \mathcal{V}(s) - \mathcal{V}(s_{k+1}) \]
\[ \mathcal{V}(t_{k+1}) = \mathcal{V}(t_k) + \int_{t_k}^{t_{k+1}} \mathcal{W}(s)ds, \quad (10) \]
\[ \mathcal{W}(s) = \begin{cases}
\dot{\mathcal{V}}(s), & t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1, \\
\Delta(\mathcal{V}(s)), & t \in \lambda_{k,L},
\end{cases}
\]
\[ \Delta(\mathcal{V}(s)) = \mathcal{V}(s) - \mathcal{V}(s_{k+1}) \]
\[ \mathcal{V}(t_{k+1}) = \mathcal{V}(t_k) + \int_{t_k}^{t_{k+1}} \mathcal{W}(s)ds, \quad (11) \]
\[ \mathcal{W}(s) = \begin{cases}
\dot{\mathcal{V}}(s), & t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1, \\
\Delta(\mathcal{V}(s)), & t \in \lambda_{k,L},
\end{cases}
\]
\[ \Delta(\mathcal{V}(s)) = \mathcal{V}(s) - \mathcal{V}(s_{k+1}) \]
\[ \mathcal{V}(t_{k+1}) = \mathcal{V}(t_k) + \int_{t_k}^{t_{k+1}} \mathcal{W}(s)ds, \quad (12) \]
\[ \mathcal{W}(s) = \begin{cases}
\dot{\mathcal{V}}(s), & t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1, \\
\Delta(\mathcal{V}(s)), & t \in \lambda_{k,L},
\end{cases}
\]
\[ \Delta(\mathcal{V}(s)) = \mathcal{V}(s) - \mathcal{V}(s_{k+1}) \]
\[ \mathcal{V}(t_{k+1}) = \mathcal{V}(t_k) + \int_{t_k}^{t_{k+1}} \mathcal{W}(s)ds, \quad (13) \]
where

\[ P_i(t) = (1 - u)P_{i,q} + uP_{i,q+1}, \]

(14)

with \( u = \frac{t}{T_{\text{min}}}(t - t_k - \theta_q). \)

Moreover,

\[ v(t_{k+1}, t, x(t)) = e^{i(t_{k+1} - t)}x^T(t)[Sx(t) + 2U^T x(t_k)] + (t_{k+1} - t) \int_{t_k}^{t} e^{i(t-s)}x^T(s)Rx(s)ds, \]

(15)

where \( \tilde{x}(t) = x(t) - x(t_k). \)

Therefore, based on time-scheduled Lyapunov functional (10), we have the following theorem.

**Theorem 1.** SDMJS (5) is mean square exponentially stable with a decay rate \( \lambda \) if there exist positive definite matrices \( P_{i,q}, q = 0, 1, \ldots, L \). \( R \) and arbitrary matrices \( S, U, N_1, M_1 \) with appropriate dimensions such that the following LMIs hold for \( T_k \in \{T_{\text{min}}, T_{\text{max}}\}, i \in I, \)

\[ \Phi_{i,l,q} + T_k \Phi_{i,l} < 0, \quad q = 0, 1, \ldots, L - 1. \]

(16)

\[ \Phi_{i,l,q+1} + T_k \Phi_{i,l} < 0, \quad q = 0, 1, \ldots, L - 1. \]

(17)

\[ \Phi_{i,l} + T_k \Phi_{i,l} < 0 \]

(18)

\[
\begin{bmatrix}
\Phi_{i,q} \\
T_k M_1^T
\end{bmatrix}
\begin{bmatrix}
T_k M_1^T \\
* -T_k e^{iT_{\text{max}}} R
\end{bmatrix}
< 0, \quad q = 0, 1, \ldots, L - 1.
\]

(19)

\[
\begin{bmatrix}
\Phi_{i,q+1} \\
T_k M_1^T
\end{bmatrix}
\begin{bmatrix}
T_k M_1^T \\
* -T_k e^{iT_{\text{max}}} R
\end{bmatrix}
< 0, \quad q = 0, 1, \ldots, L - 1.
\]

(20)

\[
\begin{bmatrix}
\Phi_{i,L} \\
T_k M_1^T
\end{bmatrix}
\begin{bmatrix}
T_k M_1^T \\
* -T_k e^{iT_{\text{max}}} R
\end{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(21)

\[ P_{j,0} - P_{i,L} < 0, \quad i \neq j. \]

(22)
Proof. Let $L$ be the week infinitesimal operator of the random process $\{x_i, r(t), t \geq 0\}$ along system (5) and we have

$$
L V_i(t) = \xi_i(t) e^{it} \left\{ (1 - u) \left[ 2 e_i^T P_{t,q} e_2 + e_i^T \sum_{j=1}^s \pi_{i,j} P_{j,q} e_1 + e_i^T \frac{1}{\delta} (P_{i,q+1} - P_{i,q}) e_1 + \lambda e_i^T P_{i,q} e_1 \right] + u \left[ 2 e_i^T P_{t,q+1} e_2 + e_i^T \sum_{j=1}^s \pi_{i,j} P_{j,q+1} e_1 + e_i^T \frac{1}{\delta} (P_{i,q+1} - P_{i,q}) e_1 + \lambda e_i^T P_{i,q+1} e_1 \right] \right\} \xi(t) 
$$

(23)

$$
L A_1(t) = e^{it} \eta(t) \left( e_i^T \lambda P_{t,L} e_1 + 2 e_i^T P_{t,L} e_2 + e_i^T \sum_{j=1}^s \pi_{i,j} P_{j,L} e_1 \right) \eta(t) 
$$

(24)

$$
L \nu(t) = \xi_i(t) e^{it} \left\{ (T_k - \sigma) \left[ \lambda (e_1 - e_3)^T S (e_1 - e_3) + 2 U e_3 \right] + e_i^T [S (e_1 - e_3) + 2 U e_3] + (e_1 - e_3)^T S e_2 + e_i^T \xi_i(t) + \lambda e_i^T \xi_i(t) \right\} \eta(t) 
$$

- \int_{t_k}^t e^{i(s-t) \xi_i(s)^T R \xi(s) ds} - \lambda (t_{k+1} - t - 1) \int_{t_k}^t e^{i(s-t) \xi_i(s)^T R \xi(s) ds} 

(25)

With the help of free-weighting matrix approach,\(^4\) for any matrices $M_i = [M_{1i} \ M_{2i} \ M_{3i}]$, the following inequality holds

$$
- \int_{t_k}^t \dot{\xi_i}^T R \dot{\xi_i} ds \leq \sigma \xi_i^T (t) M_i^T R^{-1} M_i \xi_i(t) + 2 \xi_i^T (t) M_i^T \dot{\xi_i}(t). 
$$

(26)

For any matrices $N_i = [N_{1i} \ N_{2i} \ N_{3i}]$ with appropriate dimensions,

$$
2 \xi_i^T (t) N_i^T [A_i \xi_i(t) - B_i K_i(t) x(t_k) - \dot{x}(t)] = 0. 
$$

(27)

Therefore, from (26)-(27), for $t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1$, we have

$$
L \tilde{W}(t) + \lambda (t_{k+1} - t) \int_{t_k}^t e^{i(s-t) \xi_i^T R \xi(s) ds} \leq \xi_i^T (t) \Phi_i(\sigma) \xi_i(t). 
$$

(28)

where

$$
\Phi_i(\sigma) = (1 - u) \Phi_{i,q}(\sigma) + u \Phi_{i,q+1}(\sigma), 
$$

(29)

with

$$
\Phi_{i,q}(\sigma) = \Phi_{i,t} + (T_k - \sigma) \Phi_{i,t} + \sigma \Phi_{i,t} \Phi_{i,q+1}(\sigma) = \Phi_{i,t} + (T_k - \sigma) \Phi_{i,t} + \sigma \Phi_{i,t} \Phi_{i,t} \\Phi_{i,t} \Phi_i e^{-iT_{\text{max}}} M_i^T (R)^{-1} M_i. 
$$

(30)

For $t \in \lambda_{k,t}$, we have

$$
L \tilde{W}(t) + \lambda (t_{k+1} - t) \int_{t_k}^t e^{i(s-t) \xi_i^T R \xi(s) ds} \leq \xi_i^T (t) \Phi_{i,t}(\sigma) \xi_i(t). 
$$

(33)

where

$$
\Phi_{i,t} = \Phi_{i,t} + (T_k - \sigma) \Phi_{i,t} + \sigma \Phi_{i,t}. 
$$

(34)

Note that $\Phi_{i,q}(\sigma), \Phi_{i,q+1}(\sigma), \Phi_{i,t}(\sigma)$ are affine in $\sigma$, and thus convex. It is necessary and sufficient to check it at the vertices of the set, that is, only over the finite set $\sigma \in \{0, T_k\}$. When $\sigma = 0, \Phi_{i,q}(0) < 0$ is equivalent to (16), $\Phi_{i,q+1}(0) < 0$ is equivalent to (17), $\Phi_{i,t}(0)$ is equivalent to (18). When $\sigma = T_k$, by Schur complement, $\Phi_{i,q}(T_k) < 0$ is equivalent to (19), $\Phi_{i,q+1}(T_k) < 0$ is equivalent to (20), and $\Phi_{i,t}(T_k) < 0$ is equivalent to (21). At the discrete-time sampling instants $t_k$, as explained in the work of Allerhand and Shaked,\(^4\) condition (22) guarantee that the Lyapunov function is nonincreasing.

Then, it follows from the Dynkin’s formula that

$$
\mathbb{E} \left\{ W(x(t_k), r(t_k), t_k) \right\} = W(x(0), r(0), 0) + \mathbb{E} \left\{ \int_0^{t_k} L W(x(s), r(s), s) ds \right\} \leq W(x(0), r(0), 0). 
$$

(35)
From (10), we can get

\[ W(x(0), r(0), 0) \leq \rho |x(0)|^2, \]

where

\[ \rho = \rho_1 + \rho_2, \rho_1 = \max_{i \in I} \{ \lambda_{\max}(P_i(t)) \}, \rho_2 = \max_{i \in I} \{ \lambda_{\max}(P_{i,L}) \}. \]

On the other hand, it follows from (10) that

\[ e^{\lambda t_k} \min_{i \in I} \{ \lambda_{\min}(P_i(t)), \lambda_{\min}(P_{i,L}) \} \mathbb{E} \{ |x(t_k)|^2 \} \leq \mathbb{E} \{ W(x(t_k), r(t_k), t_k) \}. \]

Therefore, we can get

\[ \mathbb{E} \{ |x(t_k)|^2 \} \leq \frac{\rho}{\gamma} e^{-\lambda t_k} |x(0)|^2, \]

where \( \gamma = e^{\lambda t_k} \min_{i \in I} \{ \lambda_{\min}(P_i(t)), \lambda_{\min}(P_{i,L}) \} \).

On the other hand, \( \forall t \in [t_k, t_{k+1}) \), the integral of system (1) can be obtained

\[ x(t) = x(t_k) + \int_{t_k}^{t} (A_i x(s) - B_i K_i x(t_k)) ds. \]

Furthermore,

\[ |x(t)| = \left| x(t_k) + \int_{t_k}^{t} (A_i x(s) - B_i K_i x(t_k)) ds \right| \]

\[ \leq |x(t_k)| + \int_{t_k}^{t} \|A_i\| |x(s)| ds + T_k - B_i K_i |x(t_k)| ds \]

\[ \leq (1 + T_{\max}) |x(t_k)| + \|A_i\| \int_{t_k}^{t} |x(s)| ds. \]

From Gronwall-Bellman inequality (Lemma 2), it is clear that

\[ |x(t)| \leq (1 + T_{\max}) |x(t_k)| e^{\|A_i\|(|t| - t_k)}. \]

Let \( \zeta = 1 + T_{\max} |x(t_k)| \), we can get

\[ |x(t)| \leq \zeta e^{\|A_i\|(|t| - t_k)} |x(t_k)|. \]

Therefore, from (37) and (41), we can obtain

\[ \mathbb{E} \{ |x(t)|^2 \} \leq \mathbb{E} \left\{ \zeta^2 e^{2\|A_i\|(|t| - t_k)} |x(t_k)|^2 \right\} \]

\[ \leq \mathbb{E} \left\{ \frac{\zeta^2 \rho}{\gamma} e^{-\lambda t_k + 2\|A_i\||t - t_k|} |x(0)|^2 \right\} \]

\[ = \mathbb{E} \left\{ \frac{\zeta^2 \rho}{\gamma} e^{\lambda (t - t_k) + 2\|A_i\||t - t_k)} e^{-\lambda t_k} |x(0)|^2 \right\} \]

\[ \leq \frac{\zeta^2 \rho}{\gamma} e^{\lambda |t| - 2\|A_i\| T_{\max}} |x(0)|^2. \]

Under Definition 1, there is

\[ \mathbb{E} \{ |x(t)|^2 \} \leq \alpha e^{-\lambda t_k} |x(0)|^2, \]

where

\[ \alpha = \frac{\zeta^2 \rho}{\gamma} e^{\lambda |t| - 2\|A_i\| T_{\max}}. \]

Therefore, SDMJS (5) is mean square exponentially stable with a decay rate \( \lambda \). This completes the proof. \( \square \)

**Remark 1.** In Theorem 1, a sufficient condition for the mean square exponential stability of the SDMJS is developed. Furthermore, we can get the quantitative relationship between sampling period and decay rate from (44)

\[ \lambda = \frac{\ln \frac{\alpha \gamma}{\zeta^2 \rho T_{\max}}}{2\|A_i\|} - 2\|A_i\|, \]

namely,

\[ \lambda = \frac{\ln \frac{\alpha \gamma}{(1 + T_{\max} |B_i K_i|)^2 \rho T_{\max}}}{2\|A_i\|} - 2\|A_i\|. \]
From (46), fixed other parameters except decay rate, and sampling period, we can find that decay rate and the sampling period have a negative correlation. Namely, the convergence speed of the system state tending to equilibrium can be improved by reducing the maximum sampling period. Let $T_{\text{min}} = T_{\text{max}}$, Theorem 1 is still effective to analyze the stability of periodic SDMJSs. For periodic SDMJSs, the convergence speed of the system state tending to equilibrium can be improved by reducing the sampling period. In fact, the negative correlation between the decay rate of the sampled-data control system and the sampling period has been revealed in the work of Zhang and Yu.51 From a new perspective, we present a new relation (46) that shows that SDMJSs also have such a negative correlation. In addition, the mean square exponential stability of SDMJSs has been investigated in the work of Shen et al.41 However, the quantitative relationship between sampling period and decay rate is not revealed in the aforementioned work.41 Numerical simulation results in Section 5 also illustrate this negative correlation.

**Remark 2.** It should be noted that (16)-(21) are affine in $T_k$ in Theorem 1, and thus convex. It is necessary and sufficient to check it at the vertices of the set, that is, only over the finite set $T_k \in \{T_{\text{min}}, T_{\text{max}}\}$. On the other hand, Theorem 1 provides a stability condition for system with aperiodic samplings. Clearly, the same conclusion remains valid for system with periodic samplings by letting $T_{\text{min}} = T_{\text{max}}$.

### 4 | TIME-DEPENDENT STATE FEEDBACK SAMPLED-DATA CONTROLLER DESIGN FOR SDMJSS

In this section, a time-dependent state feedback sampled-data controller $K(t)$ is designed to stabilize system (5). We have the following theorem.

**Theorem 2.** Given positive constants $\chi$, SDMJS (5) is mean square exponentially stable with a decay rate $\lambda$ if there exist positive definite matrices $Q_{i,q}, R_{i,q}$ and arbitrary matrices $S_{i,q}, U_{i,q}, M_{i,q}, M_{i1,q}, M_{i2,q}, M_{i3,q}, q = 0, 1, ...L, S_{i,q}, U_{i,q}, M_{i1,q}, M_{i2,q}, M_{i3,q}, q = 0, 1, ...L - 1$ with appropriate dimensions such that the following LMIs hold for $T_k \in \{T_{\text{min}}, T_{\text{max}}\}, i \in I$:

\[
\begin{bmatrix}
\Psi_{i,q} + T_k \Psi_{i,q} & e_1^T Y_{i,q} \\
* & - Y_{i,q}
\end{bmatrix} < 0, \quad q = 0, 1, ..., L - 1. \tag{47}
\]

\[
\begin{bmatrix}
\Theta_{i,q+1} + T_k \Theta_{i,q+1} & e_1^T Y_{i,q+1} \\
* & - Y_{i,q+1}
\end{bmatrix} < 0, \quad q = 0, 1, ..., L - 1. \tag{48}
\]

\[
\begin{bmatrix}
\Omega_{i,L} + T_k \Omega_{i,L} & e_1^T Y_{i,L} \\
* & - Y_{i,L}
\end{bmatrix} < 0 \tag{49}
\]

\[
\begin{bmatrix}
\Psi_{i,q} & T_k M_{i,q}^T & e_1^T Y_{i,q} \\
* & - T_k R & 0 \\
* & * & - Y_{i,q}
\end{bmatrix} < 0, \quad q = 0, 1, ..., L - 1. \tag{50}
\]

\[
\begin{bmatrix}
\Theta_{i,q+1} & T_k M_{i,q+1}^T & e_1^T Y_{i,q+1} \\
* & - T_k R & 0 \\
* & * & - Y_{i,q+1}
\end{bmatrix} < 0, \quad q = 0, 1, ..., L - 1. \tag{51}
\]

\[
\begin{bmatrix}
\Omega_{i,L} & T_k M_{i,L}^T & e_1^T Y_{i,L} \\
* & - T_k R & 0 \\
* & * & - Y_{i,L}
\end{bmatrix} < 0 \tag{52}
\]

\[Q_{i,i} - q_{i,i} < 0, \quad i \neq j, \tag{53}\]

and the time-dependent controller gain is given by

\[
K(t) = \begin{cases} 
K(t)((1 - u)Q_{i,q} + uQ_{i,q+1})^{-1}, & t \in \lambda_{k,q}, q = 0, 1, ..., L - 1 \\
K_{i,L}(Q_{i,L})^{-1}, & t \in \lambda_{k,L}
\end{cases} \tag{54}
\]
with

\[ \dot{K}(t) = (1-u)K(t) + uK(t+1), u = \frac{L}{T_{\min}}(t - t_k - \theta_q), \]

where

\[ \Psi_{1q} = \text{He} \left\{ e_1^T \left( \sigma_1 Q_{1q} e_2 \right) + e_1^T \left( \lambda Q_{1q} - 1/\delta(Q_{1q+1} - Q_{1q}) \right) e_1 \right\} 
- (e_1 - e_3)^T \left\{ \left( (1-u)S_{1q} + 2u\dot{S}_{1q} \right) (e_1 - e_3) + 2 \left( (1-u)U_{1q} + 2u\dot{U}_{1q} \right) e_3 \right\} 
+ \text{He} \left\{ e_1^\top \left( (1-u)M_{1q}^T + 2u\dot{M}_{1q}^T \right) e_1 \right\} 
+ e_1^T \left( (1-u)M_{1q+1}^T + 2u\dot{M}_{1q+1}^T \right) \} (e_1 - e_3) \right\} 
+ \text{He} \left\{ \left( e_1^T X_1 + e_2^T X_1 + e_3^T X_1 \right) \times (A(e_1 - B_k\dot{K}_{1q} e_3 - e_2)) \right\}. \]

\[ \Theta_{1q+1} = \text{He} \left\{ e_1^T \left( \sigma_1 Q_{1q+1} e_2 \right) + e_1^T \left( \lambda Q_{1q+1} - 1/\delta(Q_{1q+1} - Q_{1q}) \right) e_1 \right\} 
- (e_1 - e_3)^T \left\{ uS_{1q+1} (e_1 - e_3) + 2uU_{1q+1} e_3 \right\} 
+ \text{He} \left\{ e_1^T \left( (1-u)M_{1q+1}^T + 2u\dot{M}_{1q+1}^T \right) e_1 \right\} 
+ \text{He} \left\{ \left( e_1^T X_1 + e_2^T X_1 + e_3^T X_1 \right) \times (A(e_1 - B_k\dot{K}_{1q}+1 e_3 - e_2)) \right\}. \]

\[ \Omega_{1L} = \text{He} \left\{ e_1^T \left( \sigma_1 Q_{1L} e_2 \right) + e_1^T \left( \lambda Q_{1L} \right) e_1 \right\} 
- (e_1 - e_3)^T \left\{ \left( (1-u)S_{1L} + 2u\dot{S}_{1L} \right) (e_1 - e_3) + 2 \left( (1-u)U_{1L} + 2u\dot{U}_{1L} \right) e_3 \right\} 
+ \text{He} \left\{ e_1^T \left( (1-u)M_{1L}^T + 2u\dot{M}_{1L}^T \right) e_1 \right\} 
+ \text{He} \left\{ \left( e_1^T X_1 + e_2^T X_1 + e_3^T X_1 \right) \times (A(e_1 - B_k\dot{K}_{1L} e_3 - e_2)) \right\}. \]

Proof. Construct a Lyapunov functional candidate

\[ \dot{W}(t) = \dot{W}(t) + \nu(t, t_k + 1, x(t)), \]

(55)
where
\[
\hat{W}(t) = \begin{cases} \hat{V}(t, x(t)), t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1. \\ \hat{\lambda}(t, x(t)), t \in \lambda_{k,l}. \end{cases}
\] (56)

where
\[
\hat{V}(x(t)) = x^T(t)e^{\lambda t}Q_1^{-1}(t)x(t)
\] (57)
\[
\hat{\lambda}(x(t)) = x^T(t)e^{\lambda t}Q_1^{-1}x(t).
\] (58)

Let \( \mathcal{L} \) be the weak infinitesimal operator of the random process \( \{x_t, r(t), t \geq 0\} \) along system (5), and we have
\[
\mathcal{L}\hat{V}(x(t)) + \hat{\lambda}(t_{k+1} - t) \int_{t_k}^t e^{\lambda(s-t)}x^T(s)R \dot{x}(s)ds \leq e^{\lambda t}x^T(t)\Psi(t, \sigma)\xi(t),
\] where
\[
\Psi(t, \sigma) = \Psi_{11}(t) + (T_k - \sigma)\Phi_{12} + \sigma\Phi_{13},
\] (62)

where
\[
\Psi_{11}(t) = \mathcal{H}e \left( e_1^TQ^{-1}_1(t)e_2 \right) + e_1^T \sum_{j=1}^s \pi_{ij}Q^{-1}_j(t)e_1
\]
\[
+ e_1^T \left( \lambda Q^{-1}_1(t) - Q^{-1}_1(t)Q^{-1}_1(t)Q^{-1}_1(t) \right) e_1 + (e_1 - e_3)^T[S(e_1 - e_3) + UT_2]
\]
\[
+ \mathcal{H} \left\{ e^{-\lambda T_m} \left( e_1^TM_1^T + e_2^TM_2^T + e_3^TM_3^T \right) (e_1 - e_3) \right\}
\]
\[
+ \mathcal{H} \left\{ \left( e_1^TN_1^T + e_2^TN_2^T + e_3^TN_3^T \right) \right\} \right\} .
\] (63)

For \( t \in \lambda_{k,l} \), we can obtain
\[
\mathcal{L}\hat{V}(x(t)) + \hat{\lambda}(t_{k+1} - t) \int_{t_k}^t e^{\lambda(s-t)}x^T(s)R \dot{x}(s)ds \leq e^{\lambda t}x^T(t)\Psi(t, \sigma)\xi(t),
\] where
\[
\Psi(t, \sigma) = \Psi_{11,l} + (T_k - \sigma)\Phi_{12} + \sigma\Phi_{13},
\] (65)

where
\[
\Psi_{11,l} = \mathcal{H}e \left( e_1^TQ^{-1}_{l,l}(t)e_2 \right) + e_1^T \sum_{j=1}^s \pi_{ij}Q^{-1}_j(t)e_1
\]
\[
+ e_1^T \left( \lambda Q^{-1}_{l,l}(t) - Q^{-1}_{l,l}(t)Q^{-1}_{l,l}(t)Q^{-1}_{l,l}(t) \right) e_1 + (e_1 - e_3)^T[S(e_1 - e_3) + UT_2]
\]
\[
+ \mathcal{H} \left\{ e^{-\lambda T_m} \left( e_1^TM_1^T + e_2^TM_2^T + e_3^TM_3^T \right) (e_1 - e_3) \right\}
\]
\[
+ \mathcal{H} \left\{ \left( e_1^TN_1^T + e_2^TN_2^T + e_3^TN_3^T \right) \right\} \right\} .
\] (66)

For \( t \in \lambda_{k,q} \), let \( N_{1i} = \chi_iQ^{-1}_1(t), N_{2i} = \chi_iQ^{-1}_1(t), N_{3i} = \chi_iQ^{-1}_1(t) \). Multiply both sides of \( \Psi(t, \sigma) \) with \( \text{diag} \{Q_1(t), Q_1(t), Q_1(t) \} \). For \( t \in \lambda_{k,l} \), let \( N_{1i} = \chi_iQ^{-1}_{l,l}, N_{2i} = \chi_iQ^{-1}_{l,l}, N_{3i} = \chi_iQ^{-1}_{l,l} \). Multiply both sides of \( \Psi(t, \sigma) \) with
diag\{QL, Q_{1L}, Q_{1L}\}. Define following new variables:

\[ R_{i,q} = Q_{i,q} R_{i,q}, R_{i,q+1} = Q_{i,q+1} R_{i,q+1}, R_{i,L} = Q_{i,L} R_{i,L}, S_{i,q} = Q_{i,q} S_{i,q}, \]

\[ S_{i,q+1} = Q_{i,q+1} S_{i,q+1}, S_{i,L} = Q_{i,L} S_{i,L}, \hat{S}_{i,q} = Q_{i,q} S_{i,q+1}, \hat{U}_{i,q} = Q_{i,q} U_{i,q+1}, \]

\[ U_{i,q} = Q_{i,q} U_{i,q}, U_{i,q+1} = Q_{i,q+1} U_{i,q+1}, U_{i,L} = Q_{i,L} U_{i,L}, M_{i,q} = Q_{i,q} M_{i,q}, \]

\[ M_{2i,q} = Q_{i,q} M_{2i,q}, M_{3i,q} = Q_{i,q} M_{3i,q}, M_{1i,q+1} = Q_{i,q+1} M_{1i,q+1}, \]

\[ \hat{M}_{1i,q} = Q_{i,q} M_{1i,q}, \hat{M}_{2i,q} = Q_{i,q} M_{2i,q}, \hat{M}_{3i,q} = Q_{i,q} M_{3i,q}, \hat{M}_{1i,q+1} = Q_{i,q+1} M_{1i,q+1}, \]

\[ \hat{M}_{2i,q+1} = Q_{i,q+1} M_{2i,q}, \hat{M}_{3i,q+1} = Q_{i,q+1} M_{3i,q}, \hat{M}_{1L} = Q_{i,L} M_{1L}, \hat{M}_{2L} = Q_{i,L} M_{2L}, \]

\[ \hat{M}_{3L} = Q_{i,L} M_{3L}, \hat{K}_{i,q} = K_{i,q} Q_{i,q}, \hat{K}_{i,q+1} = K_{i,q+1} Q_{i,q+1}, \hat{K}_{i,L} = K_{i,L} Q_{i,L}. \]

Then, we have

\[ \Psi_{i}(\sigma) = (1 - u)(\Psi_{i,q}(\sigma) + Q_{i,q} + u(\Theta_{i,q+1}(\sigma) + Q_{i,q+1})) \]

\[ \Omega_{i}(\sigma) = \Omega_{i,L} + (T_k - \sigma) \Omega_{i,q} + \sigma \Psi_{i,q} + Q_{i,q}, \]

where

\[ \Psi_{i,q}(\sigma) = \Psi_{i,q} + (T_k - \sigma) \Psi_{i,q} + \sigma \Psi_{i,q} \]

\[ \Theta_{i,q+1}(\sigma) = \Theta_{i,q+1} + (T_k - \sigma) \Theta_{i,q+1} + \sigma \Psi_{i,q+1} \]

\[ Q_{i,q} = \sum_{j=1, i \neq j}^{s} \pi_{i,j} Q_{i,q} Q_{j,q}^{-1} Q_{j,q} = Y_{i,q} Y_{i,q}^{-1} Y_{i,q}, \]

\[ Q_{i,q+1} = \sum_{j=1, i \neq j}^{s} \pi_{i,j} Q_{i,q+1} Q_{j,q+1}^{-1} Q_{j,q+1} = Y_{i,q+1} Y_{i,q+1}^{-1} Y_{i,q+1}, \]

\[ Q_{i,L} = \sum_{j=1, i \neq j}^{s} \pi_{i,j} Q_{i,L} Q_{j,L}^{-1} Q_{j,L} = Y_{i,L} Y_{i,L}^{-1} Y_{i,L}, \]

\[ \Psi_{i,q} = e^{-\lambda T_{\text{max}}} M_{i,L}^{T} (R)^{-1} M_{i,q} \]

\[ \Psi_{i,q+1} = e^{-\lambda T_{\text{max}}} M_{i,L}^{T+1} (R)^{-1} M_{i,q+1} \]

\[ \Psi_{i,L} = e^{-\lambda T_{\text{max}}} M_{i,L}^{T} (R)^{-1} M_{i,L}. \]

Here, \( \Psi_{i,q}(\sigma), \Theta_{i,q+1}(\sigma), \Omega_{i,L}(\sigma) \) are affine in \( \sigma \), and thus convex. It is necessary and sufficient to check it at the vertices of the set, that is, only over the finite set \( \sigma \in \{0, T_k\} \).

When \( \sigma = 0 \), by Schur complement, \( \Psi_{i,q}(0) + Q_{i,q} < 0 \) is equivalent to (47), \( \Theta_{i,q+1}(0) + Q_{i,q+1} < 0 \) is equivalent to (48), and \( \Omega_{i}(0) < 0 \) is equivalent to (49). When \( \sigma = T_k \), by Schur complement, \( \Psi_{i,q}(T_k) + Q_{i,q} < 0 \) is equivalent to (50), \( \Theta_{i,q+1}(T_k) + Q_{i,q+1} < 0 \) is equivalent to (51), and \( \Omega_{i}(T_k) < 0 \) is equivalent to (52). At the discrete-time sampling instants \( t_k \), as explained in the work of Allerhand and Shaked, condition (53) guarantees the Lyapunov function being nonincreasing. Therefore, we can get time-dependent sampled-data controller (54) to guarantee the mean square exponential stability of system (5) with a decay rate \( \lambda \). This completes the proof.

\[ \square \]

## 5 | NUMERICAL EXAMPLES

### Example 1

Consider system (5) with following parameters:

\[ A_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, K_1 = [2 \ 0]. \]

\[ A_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, K_2 = [0 \ 1]. \]

\[ \pi_{11} = -0.2, \pi_{12} = 0.2, \pi_{21} = 0.8, \pi_{22} = -0.8. \]

For a fixed \( T_{\text{min}} = 0.3 \), we list maximum decay rate \( \lambda \) for different \( T_{\text{max}} \) in Table 1 by solving LMIs in Theorem 1.
We can see that the larger decay rate can be obtained with increase the number of dividing segments $L$. Furthermore, we can see that the maximum sampling period $T_{\text{max}}$ is negative correlated with the decay rate, which validates our statement in Remark 1. The state response of closed-loop system in this example and Markovian jump signals are shown in Figure 1. The corresponding aperiodic sampled-data control input signal and aperiodic sampling interval are shown in Figure 2. It is clear that the closed-loop systems are mean square exponentially stable.

**FIGURE 1**  Closed-loop system state response and Markovian jump signals in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

**FIGURE 2**  Sampled-data control input and aperiodic sampling intervals in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]
TABLE 2  Maximum decay rate $\lambda$ for different $T_k$ in Example 2

<table>
<thead>
<tr>
<th>Methods</th>
<th>0.5</th>
<th>0.8</th>
<th>1</th>
<th>1.2</th>
<th>NoV*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shu et al$^{11}$</td>
<td>0.968</td>
<td>0.771</td>
<td>0.660</td>
<td>0.557</td>
<td>28</td>
</tr>
<tr>
<td>Gao et al$^{12}$ (m=1)</td>
<td>1.006</td>
<td>0.796</td>
<td>0.683</td>
<td>0.590</td>
<td>39</td>
</tr>
<tr>
<td>Gao et al$^{12}$ (m=2)</td>
<td>1.022</td>
<td>0.823</td>
<td>0.714</td>
<td>0.620</td>
<td>72</td>
</tr>
<tr>
<td>Huang et al$^{13}$</td>
<td>1.197</td>
<td>1.003</td>
<td>0.839</td>
<td>0.693</td>
<td>26</td>
</tr>
<tr>
<td>Theorem 1 (L=1)</td>
<td>1.447</td>
<td>1.156</td>
<td>0.953</td>
<td>0.772</td>
<td>34</td>
</tr>
<tr>
<td>Theorem 1 (L=2)</td>
<td>2.066</td>
<td>1.717</td>
<td>1.441</td>
<td>1.143</td>
<td>51</td>
</tr>
<tr>
<td>Theorem 1 (L=3)</td>
<td>2.161</td>
<td>1.808</td>
<td>1.531</td>
<td>1.233</td>
<td>68</td>
</tr>
<tr>
<td>Theorem 1 (L=4)</td>
<td>2.211</td>
<td>1.851</td>
<td>1.569</td>
<td>1.274</td>
<td>85</td>
</tr>
<tr>
<td>Theorem 1 (L=5)</td>
<td>2.242</td>
<td>1.876</td>
<td>1.593</td>
<td>1.300</td>
<td>102</td>
</tr>
</tbody>
</table>

Example 2. Consider system (5) with following parameters$^{11-13}$:

$$A_1 = \begin{bmatrix} -0.90 & 0.50 \\ -0.32 & -0.80 \end{bmatrix}, \quad B_1 * K_1 = \begin{bmatrix} 0.50 & 0.30 \\ -0.30 & 0.20 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1.05 & 0.80 \\ -0.15 & -1.30 \end{bmatrix}, \quad B_2 * K_2 = \begin{bmatrix} -0.60 & 0.40 \\ -0.35 & 0.41 \end{bmatrix},$$

$$\pi_{11} = -0.2, \quad \pi_{12} = 0.2, \quad \pi_{21} = 0.8, \quad \pi_{22} = -0.8.$$

By the input delay approach for sampled-data systems$^{34}$ and assuming $h(t) = t - t_k$ for $t \in [t_k, t_{k+1})$, the discrete-time control input signal can be represented as follows: $u(t) = B_i * K_i \chi(t_k) = B_i * K_i \chi(t - h(t))$, $t_k \leq t < t_{k+1}$, where the sawtooth delay $h(t)$ is piecewise linear and satisfies $h(t) = 1$ for $t \neq t_k$. Therefore, the SDMJS can be as delayed MJS $\dot{x}(t) = A_i x(t) + A_{di} x(t - h(t))$, where $A_{di} = B_i * K_i$. Therefore, we can compare the maximum decay rate of delayed MJSs to illustrate the effectiveness and improvements of our method. Let $T_k = T_{\text{min}} = T_{\text{max}}$, the maximum decay rate $\lambda$ for different $T_k$ are listed in Table 2. Clearly, the proposed method yields less conservative results than those in related works.$^{11-13}$ It is also concluded from the “NoV*” (Number of Variables) in Table 2 that the reduction of conservativeness is at the expense of an increase of the overall computational complexity.

Example 3. In this example, the electrical circuit system shown in Figure 3 adopted from the works of Shen et al$^{41}$ and Sakthivel et al$^{43}$ is applied to illustrate the effectiveness of Theorem 2. In this system, the switch occupies two positions, which can be modeled as a Markov process, take two modes with the following transition rate matrix $\Pi = \begin{bmatrix} -1 & 1 \\ 0.3 & -0.3 \end{bmatrix}$. $i_2(t)$ and $u_c(t)$ denote the currents passing through the inductances and the voltage across the capacitor, respectively. Then, applying the Kirchhoff laws for $r(t) = 1, 2$, we have $\frac{di_2(t)}{dt} = \frac{u_{r(t)}}{L_r}$, $\frac{d\theta_c}{dt} = \frac{u_c(t)}{C}$. Let $x_1(t) = u_2(t)$ and $x_2(t) = i_2(t)$, then the electrical circuit of Figure 3 can be modeled as the SDMJS (5) with the following parameters:

$$A_i = \begin{bmatrix} 0 & \frac{1}{C_i} \\ -\frac{1}{L_i} & -\frac{1}{R_i} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ \frac{1}{L_i} \end{bmatrix}, \quad i = 1, 2,$$

where $C_1 = 0.5F, C_2 = 0.8F, R = 0.01\Omega, L_1 = 4H, L_2 = 8H$.

**FIGURE 3**  RLC series circuit
For the periodic sampling case, let $T_{\text{min}} = T_{\text{max}} = 0.3, \lambda = 0.5, \chi_1 = 0.1, \chi_2 = 0.3, t_k = 0.1$. When $q = 0$, let $t = 0.11$. When $q = 1$, let $t = 0.36$. The obtained sampled-data controllers with different cases by solving the LMIs in Theorem 2 are listed in Table 3. Based on the obtained periodic sampled-data controllers $K_1, q = 0, 1, 2$ and $K_2, q = 0, 1, 2$, the corresponding closed-loop system response, Markovian jump signal, sampled-data control input, and periodic sampling intervals are shown in Figures 4-9. It is shown in these figures that the proposed method is effective to guarantee the mean square exponential stability of SDMJS (5) for the periodic sampling case.

**TABLE 3** The obtained periodic sampling controllers for different $q$

<table>
<thead>
<tr>
<th>Controller</th>
<th>$q = 0$</th>
<th>$q = 1$</th>
<th>$q = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1$</td>
<td>[2.687632, 9.573]</td>
<td>[1.470527, 4.716]</td>
<td>[0.283426, 9.137]</td>
</tr>
<tr>
<td>$K_2$</td>
<td>[2.237626, 1.728]</td>
<td>[0.638626, 7.696]</td>
<td>[−0.332319, 1.442]</td>
</tr>
</tbody>
</table>

**FIGURE 4** Closed-loop system state response and Markovian jump signals in Example 3 [Colour figure can be viewed at wileyonlinelibrary.com]

**FIGURE 5** Sampled-data control input in Example 3 [Colour figure can be viewed at wileyonlinelibrary.com]
FIGURE 6 Closed-loop system state response and Markovian jump signals in Example 3 [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 7 Sampled-data control input in Example 3 [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 8 Closed-loop system state response and Markovian jump signals in Example 3 [Colour figure can be viewed at wileyonlinelibrary.com]
For the aperiodic sampling case, let $T_{\text{min}} = 0.1$, $T_{\text{max}} = 0.5$, $\lambda = 0.5$, $\chi_1 = 0.1$, $\chi_2 = 0.3$, $t_k = 0.1$. When $q = 0$, let $t = 0.11$. When $q = 1$, let $t = 0.36$. The obtained aperiodic sampled-data controllers with different cases by solving the LMIs in Theorem 2 are listed in Table 4. Based on the obtained aperiodic sampled-data controllers $K_1$, $q = 0, 1, 2$ and $K_2$, $q = 0, 1, 2$, the corresponding closed-loop system response, Markovian jump signal, aperiodic sampled-data control input, and aperiodic sampling intervals are shown in Figures 10-15. It is shown in these figures that the proposed method is effective to guarantee the mean square exponential stability of SDMJS (5) for the aperiodic sampling case.
FIGURE 11  Sampled-data control input and aperiodic sampling intervals in Example 3 [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 12  Closed-loop system state response and Markovian jump signals in Example 3 [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 13  Sampled-data control input and aperiodic sampling intervals in Example 3 [Colour figure can be viewed at wileyonlinelibrary.com]
6 | CONCLUSIONS

Using a new time-scheduled Lyapunov functional consisting of exponential-type looped-function, the mean square exponential stabilization problem of SDMJSs has been studied. Less conservative exponential stability criteria and the quantitative relationship between some system parameters, maximum sampling period, and decay rate have been proposed. A time-dependent state feedback sample-data controller has been designed based on the exponential stability criteria. At last, numerical simulation examples have illustrated the effectiveness of the proposed method and significant improvements over some existing ones.

Future works will focus on extending the proposed approach to the exponential stabilization of continuous-time SDMJSs with switching transition rates, exponential stabilization of Itô stochastic SDMJSs, exponential stabilization of networked switched systems with networked-induced delay, and packet-dropout. On the other hand, the behaviors of MJSs governing the memory property of the transition probabilities are called semi-MJSs. Extending the proposed approach to semi-MJSs is also promising, such as event-triggered fault detection filter semi-MJSs and mean-square exponentially stable with an expected decay rate for filter error dynamics of semi-MJSs.
Acknowledgements

This work was supported in part by the National Natural Science Foundation of China (61522303, U1509215, and 61621063), the Program for Changjiang Scholars and Innovative Research Team in University (IRT1208), the Changjiang Scholars Program, the Program for New Century Excellent Talents in University (NCET-13-0045), and the National Outstanding Youth Talents Support Program.

ORCID

Jian Sun http://orcid.org/0000-0001-9898-3129

REFERENCES


---

**How to cite this article:** Chen G, Sun J, Chen J. Mean square exponential stabilization of sampled-data Markovian jump systems. *Int J Robust Nonlinear Control*. 2018;28:5876–5894. [https://doi.org/10.1002/rnc.4351](https://doi.org/10.1002/rnc.4351)