Robust sampled-data control for Itô stochastic Markovian jump systems with state delay

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Summary
In this paper, the problem of robust sampled-data control for Itô stochastic Markovian jump systems (Itô SMJSs) with state delay is investigated. Using parameters-dependent Lyapunov functionals and some stochastic equations, we give stochastic sufficient stability criteria for polytopic uncertain Itô SMJSs. As a corollary, stochastic sufficient stability criteria are given for nominal Itô SMJSs. For this two cases of Itô SMJSs, based on the obtained stochastic stability criteria, their time-independent sampled-data controllers are designed, respectively. Then, for designing a time-dependent sampled-data controller for Itô SMJSs, a parameters-dependent time-scheduled Lyapunov functional is developed. New stochastic sufficient stability criteria are obtained for polytopic uncertain Itô SMJSs and nominal Itô SMJSs. Furthermore, their time-dependent sampled-data controllers are designed, respectively. Lastly, a numerical example is provided to illustrate the effectiveness of the proposed method.

KEYWORDS
Itô stochastic Markovian jump systems, parameters-dependent time-scheduled Lyapunov functional, robust sampled-data control, stability

1 | INTRODUCTION

As a special class of hybrid and stochastic systems, Markovian jump systems (MJSs) play an important role in describing many real world applications, such as economic systems,1 flight systems,2 power systems,3 communication systems,4 and networked control systems.5-7 Meanwhile, there is such a class of systems with Markovian jump parameters known as Itô stochastic Markovian jump systems (Itô SMJSs),8-10 which are also attracted widely attention. A lot of important results of Itô SMJSs have been obtained. The problem of exponential stability and exponential stabilization for Itô SMJSs with state delay was investigated in the works of Yue and Han11 and Wang et al.12 Using Nash game approach, $H_2/H_\infty$ control for Itô SMJSs was discussed in the work of Huang et al.13 Sliding-mode control problem of Itô SMJSs was discussed in related works.14-16 The fault detection filter was designed for Itô SMJSs in the work of Zhuang et al.17 Finite-time stability and stabilization were investigated for Itô SMJSs using a mode-dependent parameter approach in the work of Yan et al.18

With the development of the digital control technology, sampled-data systems where a continuous-time plant is controlled with a digital device have been paid more and more attention. Many important techniques have been applied to the sampled-data control systems. For example, input delay method,19,20 looped-functionals method,21,22 discrete-time system approach,23 and integral quadratic constraint approach.24 In addition, sampled-data control of MJSs also have
attracted much attention. Based on a piecewise continuous function method, the robust optimal sampled-data controller was designed for sampled-data MJSs in the works of Hu et al.25,26 Using the input delay method and extended dissipative definition, a desired mode-independent sampled-data controller was designed for sampled-data MJSs in the work of Shen et al.27 Based on the passivity definition and adaptive fault-tolerant mechanism, nonfragile sampled-data controller was designed in the works of Wu et al.28 and Sakthivel et al.29 for MJSs with aperiodic sampling. However, as far as we know, there are few results about robust sampled-data control for Itô SJMSs up to now. Similar to sampled-data feedback control of continuous-time systems, discrete-time feedback control of continuous-time stochastic system with Markovian switching is discussed in the work of Mao.30 Under dwell-time constraints, stability analysis and stabilization of stochastic sampled-data systems are investigated in the work of Briat.31 From the earlier discussion, we can find that the sampled-data control of polytopic uncertain Itô SJMSs with state delay was rarely involved. Moreover, how to design a time-dependent sampled-data controller for polytopic uncertain Itô SJMSs is a very interesting topic, which has not fully addressed.

In this paper, the problem of robust sampled-data control for Itô SJMSs with state delay is investigated. The main contributions of this paper can be summarized in four aspects. Firstly, using parameters-dependent Lyapunov functional and some related stochastic equations, we give stochastic sufficient stability criteria for polytopic uncertain Itô SJMSs with state delay and nominal Itô SJMSs with state delay. Secondly, on the basis of the stochastic stability criteria, time-independent sampled-data state feedback controllers are designed. Thirdly, inspired by the region-dividing technique, new stochastic stability criteria for polytopic uncertain Itô SJMSs with state delay and nominal Itô SJMSs with state delay are proposed by constructing new parameters-dependent time-scheduled Lyapunov functionals. Fourthly, the time-dependent state feedback sampled-data controllers are designed for polytopic uncertain Itô SJMSs with state delay and nominal Itô SJMSs with state delay based on the new stochastic stability criteria.

Notation. Throughout this paper, I denotes the identity matrix with appropriate dimension; $M^T$ represents the transpose of the matrix $M$; $X \succ 0$ ($\succcurlyeq 0$) means that $X$ is a symmetric positive-definite (positive-semidefinite) matrix; $\mathbb{R}$ denotes the set of real numbers; $\mathbb{R}^n$ denotes the set of $n$-dimensional real vector; $\mathbb{R}_+$ refers to the set of all nonnegative real numbers; $\mathbb{C}^{2,1}(\mathbb{R}_+; \mathbb{R}^{n \times n})$ represents the family of all real-valued functions $V(x(t))$ defined on $\mathbb{R}^n \times \mathbb{R}_+$, which are continuously twice differentiable in $x \in \mathbb{R}^n$ and once differentiable in $t \in \mathbb{R}_+$; $\mathbb{N} = \{1, 2, \ldots \}$, $\mathbb{R}^{n \times n}$ is the set of $n \times n$ real symmetric matrices; $\mathbb{L}_2[0, \infty)$ stands for the space of square integrable functions on $[0, \infty)$; $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation; $\text{He}(A) = A + A^T$; and $\text{diag}(...)$ denotes a block-diagonal matrix. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

### 2 PROBLEM FORMULATION AND PRELIMINARIES

Consider a polytopic uncertain Itô SJMS with state delay

$$
\begin{aligned}
\dot{x}(t) &= \begin{bmatrix}
A_{\theta}(\theta) & A_{\delta\theta}(\theta)
\end{bmatrix} x(t) + B_{\theta}(\theta) w(t) + E_{\theta}(\theta) \Phi(t) + F_{\theta}(\theta) \Phi(t)
\end{aligned}
$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system; $u(t) \in \mathbb{R}^m$ is the control vector of the system; $w(t)$ is a scalar describing Brownian motion in the complete probability space $(\Omega, \mathcal{F}, P)$, satisfying $\mathbb{E}\{dw(t)\} = 0$ and $\mathbb{E}\{w^2(t)\} = dt$; $\Phi(t)$ is a vector-valued initial continuous function defined on the interval $[-h, 0]$; and $r(t)$ is a continuous time Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{N} = \{1, 2, \ldots, s\}$ with transition probability matrix $\Pi \triangleq [\pi_{ij}]_{s \times s}$ given by

$$
Pr\{r(t + h) = j | r(t) = i\} = \begin{cases}
\pi_{ij} h + o(h), i \neq j, \\
1 + \pi_{ii} h + o(h), i = j,
\end{cases}
$$

where $h > 0$, $\lim_{h \to 0} (o(h)/h) = 0$. Here, $\pi_{ij} \geq 0$ is the transition rate from mode $i$ at time $t$ to mode $j$ at time $t + h$ if $i \neq j$ and $\pi_{ii} = -\sum_{j=1, j \neq i}^{n} \pi_{ij}$.

Matrices $A_{\theta}(\theta), A_{\delta\theta}(\theta), B_{\theta}(\theta), E_{\theta}(\theta), F_{\theta}(\theta)$ are with the following polytope presentation:

$$
\theta \in \mathcal{G} := \text{co}\{\theta_1, \theta_2, \ldots, \theta_m\}
$$
\( H_{r(t)}(\theta) \triangleq \{ A_{r(t)}(\theta), A_{d_{r(t)}}(\theta), B_{r(t)}(\theta), E_{r(t)}(\theta), F_{r(t)}(\theta) \} \in \mathcal{G} \)

\[
\mathcal{G} \triangleq \left\{ H_{r(t)}(\theta) \big| H_{r(t)}(\theta) = \sum_{a=1}^{m} \theta_a H_{r(t)}^{(a)} : \sum_{a=1}^{m} \theta_a = 1, \theta_a \geq 0 \right\}
\]

\[
H_{r(t)}^{(a)} \triangleq \left\{ A_{r(t)}^{(a)}, A_{d_{r(t)}}^{(a)}, B_{r(t)}^{(a)}, E_{r(t)}^{(a)}, F_{r(t)}^{(a)} \right\},
\]

where \( \theta \) varies in a polytope of vertices \( \theta_1, \theta_2, \ldots, \theta_m \), the symbol ‘co’ denotes the convex hull, \( \mathcal{G} \) is a given convex bounded polyhedral domain described by \( r \) vertices, and \( H_{r(t)}^{(a)} \) represents the vertices of the polytope. For the sake of simplicity, we denote \( A_{r(t)}^{(a)} = A_{r(t)}^{(a)} \) and the other symbols are similarly denoted.

For convenience, we define new state variables \( f_i(t), g_i(t) \) as follows:

\[
f_i(t) = A_{r(t)}(\theta)x(t) + A_{d_{r(t)}}(\theta)x(t-h) + B_{r(t)}(\theta)u(t)
\]

\[
g_i(t) = E_{r(t)}(\theta)x(t) + E_{d_{r(t)}}(\theta)x(t-h).
\]

Therefore, system (1) becomes

\[
dx(t) = f_i(t)dt + g_i(t)dw(t).
\]

**Definition 1.** (See the works of Skorokhod\(^8\) and Briat\(^3\)) System (1) is said to be mean-square stable, if for any \( \varepsilon > 0 \), there exists a scalar \( \delta(\varepsilon) > 0 \) such that \( E(|x(t)|^2) < \varepsilon \), \( \forall t \geq 0 \), when \( \text{sup}_{-h \leq s \leq 0} E(|\phi(s)|^2) < \delta(\varepsilon) \). Additionally, if \( \lim_{t \to \infty} E(|x(t)|^2) = 0 \) holds for any initial condition, then system (1) is said to be mean-square asymptotically stable.

Moreover, system (1) is said to be mean-square asymptotically stable if and only if \( \lim_{k \to \infty} E(|x(t_k^+)|^2) = 0 \), where \( x(t_k^+) = \lim_{t \to t_k} x(t) \) holds for any initial condition.

**Lemma 1.** (See other works\(^8-10\)) Let \( V(x, t, i) \in C^{2,1} \in (\mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{N}; \mathbb{R}_+) \), which is continuously twice differentiable in \( x \) and once differentiable in \( t \), then its stochastic differential along system (1) is given by

\[
dV(x, t, i) = \mathcal{L}V(x, t, i) + V_x(x, t, i)g(x, t, i)dw(t),
\]

where the operator \( \mathcal{L}V(x, t, i) \) is defined as

\[
\mathcal{L}V(x, t, i) = V_t(x, t, i) + V_x(x, t, i)f(x, t, i) + \frac{1}{2} \text{tr} \left\{ g^T(x, t, i)V_{xx}(x, t, i)g(x, t, i) \right\}
\]

\[
+ \sum_{j=1}^{s} \sigma_{ij}V(x, t, j),
\]

with

\[
V_t(x, t, i) = \frac{\partial V(t, x, i)}{\partial t},
\]

\[
V_x(x, t, i) = \left( \frac{\partial V(t, x, i)}{\partial x_1}, \frac{\partial V(t, x, i)}{\partial x_2}, \ldots, \frac{\partial V(t, x, i)}{\partial x_n} \right),
\]

\[
V_{xx}(x, t, i) = \left( \frac{\partial^2 V(t, x, i)}{\partial x_j \partial x_k} \right)_{n \times n}.
\]

**Lemma 2.** (See the work of Skorokhod\(^a\)) Let \( g(s) \in \mathbb{R}^n \) be a continuous real vector functional; \( w(t) \) be a one-dimensional Brownian motion, satisfying \( E\{dw(t)\} = 0 \) and \( E\{dw^2(t)\} = dt \); and \( M \) be a positive-definite matrix. Then, for any scalars \( a \) and \( b \) \((a < b)\), the following equality holds:

\[
\int_a^b g^T(s)Mg(s)ds = \mathbb{E}\left( \left( \int_a^b g(s)dw(s) \right)^T M \left( \int_a^b g(s)dw(s) \right) \right).
\]
Lemma 3. (See the work of Skorokhod⁸) If \( x(t) \) is the solution of system (1) and \( N \) is any matrix with compatible dimension, then

\[
\mathbb{E} \left( x^T(t - h)N \left[ \int_{t-h}^t g_i(s)dw(s) \right] \right) = 0.
\]  

(11)

Lemma 4. (See the work of Gu et al³²) For a given \( n \times n \) positive matrix \( R \) and for all continuous functions \( x \in [a, b] \to \mathbb{R}^n \), the following inequality holds:

\[
\int_a^b x^T(u)Rx(u)du \geq \frac{1}{b-a} \int_a^b x^T(u)duR \int_a^b x(u)du.
\]  

(12)

3.1 Stochastic stability analysis for Itô SMJSs

In this section, for simplicity of vector and matrix representation, some reconstructed vectors and matrices are denoted by

\[
\xi_i^T(t) = \begin{bmatrix} x^T(t) & x^T(t - h) & x^T(t_k) & \int_{t-h}^t f_i^T(s)ds & \int_{t_k}^t f_i^T(s)ds & f_i^T(t) \end{bmatrix}
\]  

(13)

\[
e_i = \begin{bmatrix} 0_{nx(1-1)nx} & I_{nxn} & 0_{nx(6-1)nx} \end{bmatrix} \in \mathbb{R}^{nx6n}, i = 1, 2, \ldots, 6.
\]  

(14)

The state of (1) is sampled when \( t = t_k \), where \( \{t_k\} \) represents a set of the sampling instants and satisfies

\[
0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots, t_k \in \mathbb{R}^+, \forall k \in \mathbb{N}.
\]  

(15)

The control input is a piecewise constant signal and is given by

\[
u(t) = -K_i^{(a)}x(t_k), \forall t \in [t_k, t_{k+1}).
\]  

(16)

Here, \( K_i^{(a)} \) is a state feedback Markovian jump mode-dependent controller gain matrix. The sampling does not change jump action of Markovian process. The sampling interval \( T_k \) is denoted by \( T_k = t_{k+1} - t_k \) and satisfies \( 0 < T_{\min} \leq T_k \leq T_{\max} \).

3.1 Stochastic stability analysis for Itô SMJSs

Now, we have the following theorem to guarantee that system (1) is mean-square asymptotically stable.

**Theorem 1.** Itô SMJS (1) is mean-square asymptotically stable if there exist positive-definite matrices

\[
P_i^{(a)}, Q^{(a)}, Z^{(a)}, S^{(a)}, R^{(a)}, \text{any matrices } U_i \text{ with appropriate dimensions such that the following linear matrix inequalities (LMIs) hold for } i \in \mathcal{N}, a = 1, 2, \ldots, m:
\]

\[
\begin{bmatrix}
\phi_i^{(a)} & H_i^{(a)} & 0 & 0 & 0 & 0 \\
-1 & -P_i^{(a)} & -Z^{(a)} & 0 & 0 & 0 \\
* & * & 0 & -hZ^{(a)} & 0 & 0 \\
* & * & * & * & -S^{(a)}
\end{bmatrix} < 0,
\]  

(17)
where
\[ T_k \in \{ T_{\text{min}}, T_{\text{max}} \}, \]
\[ \phi_i^{(a)} = \eta_i^{(a)} + \eta_i^{(a)} + \eta_i^{(a)} + \eta_i^{(a)} + \eta_i^{(a)}, \]
\[ \eta_i^{(a)} = \text{He} \left\{ e_i^T P_i^{(a)} A_i x + e_i^T P_i^{(a)} A_i \mathbf{e}_2 - e_i^T P_i B_i^{(a)} K_i^{(a)} \mathbf{e}_3 \right\} + e_i^T \left( \sum_{j=1}^{s_j} \pi_j P_j^{(a)} \right) \mathbf{e}_1, \]
\[ \eta_i^{(a)} = e_i^T Q_i \mathbf{e}_1 - e_i^T Q_i \mathbf{e}_2, \]
\[ \eta_i^{(a)} = -\left( e_i^T - e_i^T \right) S_i^{(a)} \mathbf{e}_1 - e_i^T Z_i^{(a)} \mathbf{e}_2, \]
\[ \eta_i^{(a)} = -\left( e_i^T - e_i^T \right) S_i^{(a)} \mathbf{e}_1 + e_i^T Z_i^{(a)} \mathbf{e}_2, \]
\[ \eta_i^{(a)} = \frac{1}{T_{\text{max}}} e_i^T R_i \mathbf{e}_5, \]
\[ \eta_i^{(a)} = \text{He} \left\{ e_i^T \left( A_i^{(a)} \right)^T U_i^T \mathbf{e}_6 + e_i^T \left( A_i^{(a)} \right)^T U_i^T \mathbf{e}_6 - e_i^T \left( K_i^{(a)} \right)^T B_i^{(a)} \right\} + U_i^T \mathbf{e}_6 - e_i^T \left( U_i^T + U_i \right) \mathbf{e}_6, \]
\[ H_i^{(a)} = \left[ E_i^{(a)} \ E_i^{(a)} \ 0 \ 0 \ 0 \right]. \]

**Proof.** Construct a Lyapunov-Krasovskii functional candidate

\[ W(x_i, i, t) = \sum_{k=1}^{2} V_k(x(x_i, i, t)) + \theta(t_{k+1}, t, x_i), \quad (18) \]

where

\[ V_1(x_i, i, t) = \sum_{a=1}^{m} x^T(t) \theta_a P_i^{(a)} x(t) \]

\[ V_2(x_i, i, t) = \sum_{a=1}^{m} \int_{t-h}^{t} x^T(s) \theta_a Q_i^{(a)} x(s) ds + \sum_{a=1}^{m} \int_{t-h}^{t} \text{Tr} \left\{ \left( g_i^T(s) \right) \theta_a Z_i^{(a)} g_i(s) \right\} ds \]

\[ \theta(t_{k+1}, t, x_i) = \sum_{a=1}^{m} \left( t_{k+1} - t \right) x_i^T(t) \theta_a S_i^{(a)} x_i(t) + \sum_{a=1}^{m} \left( t_{k+1} - t \right) \int_{t_h}^{t} \left( f_i^T(s) \right) \theta_a R_i^{(a)} f_i(s) ds, \]

with \( \bar{x}(t) = x(t) - x(t_k) \).

Let \( \mathcal{L} \) be the weak infinitesimal operator of the random process \( \{ x_i, r(t), t \geq 0 \} \) along system (1) and we have

\[ \mathcal{L} V_1(x_i, i, t) = \sum_{a=1}^{m} \theta_a \left\{ 2x^T(t) P_i^{(a)} f_i(t) + \text{Tr} \left\{ g_i^T(t) P_i^{(a)} g_i(t) \right\} + x^T(t) \left( \sum_{j=1}^{s_i} \pi_i P_j^{(a)} \right) x(t) \right\} \]

\[ = \sum_{a=1}^{m} \theta_a \left\{ x_i^T(t) \eta_i^{(a)} x_i(t) \right\} + \sum_{a=1}^{m} \theta_a \left\{ \text{Tr} \left\{ g_i^T(t) P_i^{(a)} g_i(t) \right\} \right\} \]

\[ \mathcal{L} V_2(x_i, i, t) = \sum_{a=1}^{m} \theta_a \left\{ x^T(t) Q_i^{(a)} x(t) - x^T(t - h) Q_i^{(a)} x(t - h) + \text{Tr} \left\{ g_i^T(t) h Z_i^{(a)} g_i(t) \right\} - \int_{t-h}^{t} \text{Tr} \left\{ g_i^T(s) Z_i^{(a)} g_i(s) \right\} ds \right\} \]

\[ = \sum_{a=1}^{m} \theta_a \left\{ x_i^T(t) \eta_i^{(a)} x_i(t) + \text{Tr} \left\{ g_i^T(t) h Z_i^{(a)} g_i(t) \right\} \right\} - \sum_{a=1}^{m} \theta_a \left\{ \int_{t-h}^{t} \text{Tr} \left\{ g_i^T(s) Z_i^{(a)} g_i(s) \right\} ds \right\} \]

\[ \mathcal{L} \theta(t_{k+1}, t, x_i) = \sum_{a=1}^{m} \theta_a \left\{ - \left( x^T(t) - x^T(t_k) \right) S_i^{(a)} \left( x(t) - x(t_k) \right) + 2(t_{k+1} - t) \times \left( x^T(t) - x^T(t_k) \right) S_i^{(a)} f_i(t) \right. \]

\[ + \left. \text{Tr} \left\{ (g_i(t))^{T} S_i^{(a)} g_i(t) \right\} - \int_{t_h}^{t} f_i^T(s) R_i^{(a)} f_i(s) ds + (t_{k+1} - t) f_i^T(t) R_i^{(a)} f_i(t) \right\} \]

\[ \leq \sum_{a=1}^{m} \theta_a \left\{ x_i^T(t_{k+1}) \eta_i^{(a)} x_i(t) + g_i^T(t) S_i^{(a)} g_i(t) \right\} - \sum_{a=1}^{m} \theta_a \left\{ \int_{t_h}^{t} f_i^T(s) R_i^{(a)} f_i(s) ds \right\}. \quad (24) \]
Using Lemma 2, we can get
\[
\mathbb{E} \left( \sum_{a=1}^{m} \left( \int_{t-h}^{t} g_i^T(s) \theta_a Z^{(a)}(s) \, ds \right) \right) = \mathbb{E} \left( \sum_{a=1}^{m} \left( \int_{t-h}^{t} g_i^T(s) \, dw(s) \right) \theta_a Z^{(a)} \left( \int_{t-h}^{t} g_i(s) \, dw(s) \right) \right). \tag{25}
\]
Integrating the system (1) on both sides from \( t - h \) to \( t \), we can have
\[
x(t) - x(t - h) = \int_{t-h}^{t} f_i(s) \, ds + \int_{t-h}^{t} g_i(s) \, dw(s), t \geq h.
\tag{26}
\]
Therefore, with Lemma 3, we can get
\[
\mathbb{E} \left( \sum_{a=1}^{m} \theta_a \int_{t-h}^{t} g_i^T(s) Z^{(a)} g_i(s) \, ds \right) = \mathbb{E} \left( \sum_{a=1}^{m} \theta_a \left( x(t) - x(t - h) - \int_{t-h}^{t} f_i(s) \, ds \right)^T Z^{(a)} \left( x(t) - x(t - h) - \int_{t-h}^{t} f_i(s) \, ds \right) \right) \\
= \mathbb{E} \left( \sum_{a=1}^{m} \theta_a \left( x(t) - \int_{t-h}^{t} f_i(s) \, ds \right)^T Z^{(a)} \left( x(t) - \int_{t-h}^{t} f_i(s) \, ds \right) \right) \\
- 2x(t - h)^T Z^{(a)} \left( x(t) - \int_{t-h}^{t} f_i(s) \, ds \right) + x^T(t - h) Z^{(a)} x(t - h) \\
= \mathbb{E} \left( \sum_{a=1}^{m} \theta_a \left( x(t) - \int_{t-h}^{t} f_i(s) \, ds \right)^T Z^{(a)} \left( x(t) - \int_{t-h}^{t} f_i(s) \, ds \right) \right) \\
- 2x(t - h)^T Z^{(a)} \left( x(t) - h + \int_{t-h}^{t} g_i(s) \, dw(s) \right) + x^T(t - h) Z^{(a)} x(t - h) \\
= \mathbb{E} \left( \sum_{a=1}^{m} \theta_a \left( x(t) - \int_{t-h}^{t} f_i(s) \, ds \right)^T Z^{(a)} \left( x(t) - \int_{t-h}^{t} f_i(s) \, ds \right) - x^T(t - h) Z^{(a)} x(t - h) \right) \\
= \mathbb{E} \left( \sum_{a=1}^{m} \theta_a \left( \xi_i^T(t) \left( -\xi_i^{(a)}(t) \right) \right) \right). \tag{27}
\]
From Lemma 4, we have
\[
- \sum_{a=1}^{m} \theta_a \int_{t_i}^{t} f_i^T(s) R^{(a)} f_i(s) \, ds \\
\leq \sum_{a=1}^{m} \theta_a \left( -\frac{1}{t - t_i} \left( \int_{t_i}^{t} f_i(s) \, ds \right)^T R^{(a)} \int_{t_i}^{t} f_i(s) \, ds \right) \\
\leq \sum_{a=1}^{m} \theta_a \left( -\frac{1}{T_{\text{max}}} \left( \int_{t_i}^{t} f_i(s) \, ds \right)^T R^{(a)} \int_{t_i}^{t} f_i(s) \, ds \right) \\
= \sum_{a=1}^{m} \theta_a \left( \xi_i^{T}(t) \xi_i^{(a)}(t) \right). \tag{28}
\]
On the other hand, for any free weighting matrix \( U_i \), the following equation holds:
\[
2 f_i^T(t) U_i \left( \sum_{a=1}^{m} \theta_a \left( A_i^{(a)} x(t) + A_k^{(a)} x(t - h) - B_i^{(a)} K^{(a)} x(t_k) \right) \right) - f_i(t) \right) = 0. \tag{29}
\]
namely,
\[
\sum_{a=1}^{m} \theta_a \xi_i^{T}(t) \eta_i^{(a)}(t) = 0. \tag{30}
\]
Therefore, from (22)-(30), we can get
\[
\mathbb{E} \left( \mathcal{L}W(t) \right) = \mathbb{E} \left( \sum_{a=1}^{m} \xi_i^{T}(t) \phi_i^{(a)}(t) \right), \tag{31}
\]
where \( \phi_i^{(a)} = \phi_i^{(a)} + (H_i^{(a)})^T P_i^{(a)} H_i^{(a)} + h(H_i^{(a)})^T Z^{(a)} H_i^{(a)} + (H_i^{(a)})^T S^{(a)} H_i^{(a)}. \)
By Schur complement, when $\phi_t^{(a)} < 0$, we can obtain (17). Therefore, we can prove that system (1) is mean-square asymptotically stable under Definition 1 in the works of Skorokhod and Briat. This completes the proof.

Consider the following nominal Itô SMJSs:

$$
\begin{align*}
\dot{x}(t) &= \left[ A_{\epsilon(t)}x(t) + A_{d\epsilon(t)}x(t-h) + B_{\epsilon(t)}u(t) \right] dt \\
&+ \left[ E_{\epsilon(t)}x(t) + E_{d\epsilon(t)}x(t-h) \right] dw(t), \\
x(t) &= \phi(t), \ t \in [-h, 0].
\end{align*}
$$

(32)

For convenience, we define new state variable $f_i(t), g_i(t)$

$$
\begin{align*}
f_i(t) &= A_{\epsilon(t)}x(t) + A_{d\epsilon(t)}x(t-h) + B_{\epsilon(t)}u(t) \\
g_i(t) &= E_{\epsilon(t)}x(t) + E_{d\epsilon(t)}x(t-h).
\end{align*}
$$

(33)  (34)

Therefore, system (32) becomes

$$
\dot{x}(t) = f_i(t) dt + g_i(t) dw(t).
$$

(35)

We can get the mean-square asymptotically stability criteria for nominal Itô SMJSs (32) from Theorem 1. It is summarized as Corollary 1.

**Corollary 1.** Itô SMJS (32) is mean-square asymptotically stable if there exist positive-definite matrices $P_i, Q, Z, S, R$, any matrices $U_i$ with appropriate dimensions such that the following LMIs hold for $i \in \mathcal{N}$:

$$
\begin{align*}
\begin{bmatrix}
\phi_i & H_i^T P_i & hH_i^T Z & H_i^T S \\
* & -P_i & 0 & 0 \\
* & * & -hZ & 0 \\
* & * & * & -S
\end{bmatrix} &< 0,
\end{align*}
$$

(36)

where

$T_k \in \{ T_{\min}, T_{\max} \}$,

$\phi_i = \eta_i + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_6$,

$\eta_i = \text{He} \left\{ e_1^T P_i A e_1 + e_1^T P_i A_{di} e_2 - e_1^T P_i B_i K_i e_3 \right\} e_1 + e_1^T \left\{ \sum_{j=1}^r \pi_{ij} P_j \right\} e_1$,

$\eta_2 = e_1^T Q e_1 - e_1^T Q e_2$,

$\eta_3 = - (e_1^T - e_3^T) S (e_1 - e_3) + \text{He} \left\{ T_k (e_1^T - e_3^T) S e_6 \right\} + T_k e_2^T R e_6$,

$\eta_4 = - (e_1^T - e_3^T) Z (e_1 - e_4) + e_2^T Z e_2$,

$\eta_5 = - \frac{1}{T_{\max}} e_3^T R e_5$,

$\eta_6 = \text{He} \left\{ e_1^T A_i^T U_i^T e_6 + e_1^T A_{di}^T U_i^T e_6 - e_1^T K_i^T B_i^T U_i^T e_6 - e_1^T (U_i^T + U_i) e_6 \right\}$,

$H_i = [E_i \ E_{di} \ 0 \ 0 \ 0]$

**Proof.** Construct a Lyapunov-Krasovskii functional candidate

$$
W(x_i, i, t) = \sum_{k=1}^2 \tilde{V}_k (x(x_i, i, t)) + \tilde{\theta}(t_{k+1}, t, x_i),
$$

(37)

where

$$
\begin{align*}
\tilde{V}_1(x_i, i, t) &= x^T(t) P_i x(t) \\
\tilde{V}_2(x_i, i, t) &= \int_{t-h}^{t} x^T(s) Q x(s) ds + \int_{t-h}^{t} \int_{t+\theta}^{t} \text{Tr} \left\{ g_i^T(s) Z g_i(s) \right\} ds \\
\tilde{\theta}(t_{k+1}, t, x_i) &= (t_{k+1} - t) \tilde{x}^T(t) S \tilde{x}(t) + (t_{k+1} - t) \int_{t_h}^{t} f_i^T(s) R f_i(s) ds
\end{align*}
$$

(38)  (39)  (40)

with $\tilde{x}(t) = x(t) - x(t_k)$.

Then, following the similar line as in the proof of Theorem 1, the proof can be completed.
3.2 Time-independent sampled-data controller design for Itô SMJSs

For system (1), we will give the following theorem to design a time-independent sampled-data controller to guarantee the mean-square asymptotical stability.

**Theorem 2.** Given positive constant \( \bar{\lambda}_i \), Itô SMJS (1) is mean-square asymptotically stable with time-independent sampled-data controller gain matrix \( K_i^{(a)} = \hat{K}_{i}^{(a)}(Q_i^{(a)})^{-1} \) if there exist positive-definite matrices \( Q_i^{(a)}, \mathcal{Q}_i^{(a)}, Z_i^{(a)}, S_i^{(a)}, R_i^{(a)}, \mathcal{S}_i^{(a)}, S_i^{(a)}, S_i^{(a)} \), with appropriate dimensions such that the following LMIs hold for \( i \in \mathcal{N}, a = 1, 2, \ldots, r \):

\[
\begin{bmatrix}
\varphi_i^{(a)}(Q_i^{(a)}(H_i^{(a)})^T \mathcal{H}_i^{(a)})^T hQ_i^{(a)}(H_i^{(a)})^T Q_i^{(a)}(H_i^{(a)})^T \ & e^T e_i^{(a)}
\end{bmatrix} < 0,
\]

where

\[ T_k \in \{ T_{\text{min}}, T_{\text{max}} \}, \]

\[ \varphi_i^{(a)}(Q_i^{(a)}(H_i^{(a)})^T \mathcal{H}_i^{(a)}) = \mathcal{Q}_i^{(a)} + \mathcal{Z}_i^{(a)} + \mathcal{S}_i^{(a)} \]

From Theorem 2, time-independent sampled-data feedback controller of nominal Itô SMJSs (32) is obtained. It is summarized as Corollary 2.
**Corollary 2.** Given positive constant $\lambda_i$, Itô SMJS (32) is mean-square asymptotical stable with time-independent sampled-data controller gain matrix $K_i = \hat{K}_iQ_i^{-1}$ if there exist positive-definite matrices $Q_i, \bar{Q}_i, Z_i, S_i, R_i, T, S$ with appropriate dimensions such that the following LMIs hold for $i \in \mathcal{N}$:

$$
\begin{bmatrix}
\varphi_i & Q_iH_i^T & hQ_iH_i^T & Q_iH_i^T e_i^T \Lambda_{i1} \\
* & -Q_i & 0 & 0 \\
* & * & -hZ^{-1} & 0 \\
* & * & * & -S^{-1} \\
* & * & * & * & -\Lambda_{2i}
\end{bmatrix} < 0,
$$

(43)

where

$$
T_k \in \{ T_{\min}, T_{\max} \},
$$

$$
\varphi_i = \zeta_{i1} + \zeta_{i2} + \zeta_{i3} + \zeta_{i4} + \zeta_{i6},
$$

$$
\zeta_{i1} = \mathbf{H} \left\{ e_i^TA_iQ_i e_1 + e_i^T A_i Q_i e_2 - e_i^T B_i \hat{K}_i e_3 \right\} + e_i^T \pi_i Q_i e_1,
$$

$$
\zeta_{i2} = e_i^T Q_i e_1 - e_i^T \bar{Q}_i e_2,
$$

$$
\zeta_{i3} = -(e_i^T - e_i^T) \hat{S}_i (e_1 - e_3) + \mathbf{H} \left\{ T_k (e_i^T - e_i^T) \hat{S}_i e_6 \right\} + T_k e_i^T R_i e_6,
$$

$$
\zeta_{i4} = -(e_i^T - e_i^T) \tilde{Z}_i (e_1 - e_4) + e_i^T \tilde{Z}_i e_2,
$$

$$
\zeta_{i5} = -\frac{1}{T_{\max}} T_k e_i^T \tilde{R}_i e_5,
$$

$$
\zeta_{i6} = \mathbf{H} \left\{ e_i^T A_i^T Q_i^T e_6 + e_i^T \lambda_iA_i^T e_6 - e_i^T \lambda_i A_i^T e_6 - \lambda_i e_i^T (Q_i + \bar{Q}_i) e_6 \right\},
$$

$$
Q_{ij} = \sum_{j=1}^{s} \pi_{ij} Q_i Q_j^{-1} Q_i = \Lambda_{i1} \Lambda_{i2}^{-1} \Lambda_{i1}^T,
$$

$$
\Lambda_{i1} = \left[ \sqrt{\rho_{i1}}, \ldots, \sqrt{\rho_{iL-1}}, \sqrt{\rho_{IL}}, \sqrt{\rho_{1}}, \ldots, \sqrt{\rho_{iL}} \right],
$$

$$
\Lambda_{i2} = \text{diag} \{ Q_1, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_i \}.
$$

### 4. NEW STOCHASTIC STABILITY CRITERIA AND TIME-DEPENDENT ROBUST SAMPLED-DATA CONTROLLER DESIGN FOR ITÔ SMJSS

#### 4.1 New stability criteria for Itô SMJSs based on a time-scheduled Lyapunov functional

In this section, for designing a time-dependent sampled-data controller, we derive new stochastic stability criteria based on a time-scheduled Lyapunov-Krasovskii functional. For system (1), the control input is a piecewise constant signal and is given by

$$
u(t) = -K_i^{(\alpha)}(t)x(t_k), \ \forall t \in [t_k, t_{k+1}).
$$

(44)

The sampling interval $T_k$ is denoted by $T_k = t_{k+1} - t_k$ and satisfying $0 < T_{\min} \leq T_k \leq T_{\max}$. For the sampling-data interval $[t_k, t_k + T_{\min})$, we divided it into $L$ equal segments. The length of every segment is $\frac{T_{\min}}{L}$. Setting $\theta_0 = 0, \theta_q = q \frac{T_{\min}}{L}, \lambda_{k,q} = [t_k + \theta_q, t_k + \theta_{q+1}), q = 0, 1, \ldots, L - 1, \lambda_{k,L} = [t_k + T_{\min}, t_{k+1})$. There are $\bigcup_{q=0}^{L-1} \lambda_{k,q} = [t_k, t_k + T_{\min})$ and $\lambda_{k,0} \bigcap \lambda_{k,m} = \emptyset$. Let $P_{i,q} = P_i(t_k + \theta_q)$, since $P_i(t)$ is piecewise linear and using a linear interpolation formula, for $0 \leq u \leq 1$, we have the Lyapunov-Krasovskii functional candidate

$$
W(x_i, t) = \tilde{V}_1(x_i, i, t) + V_2(x_i, i, t) + \theta(t_{k+1}, t, x_i),
$$

(45)

where

$$
\tilde{V}_1(x_i, i, t) = \sum_{a=1}^{m} \theta_a x^T(t)P_i^{(\alpha)}(t)x(t),
$$

(46)

with

$$
P_i^{(\alpha)}(t) = \begin{cases} 
P_i^{(\alpha)}(u), & t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1. \\
P_i^{(\alpha)}(t), & t \in \lambda_{k,L},
\end{cases}
$$

(47)

where

$$
P_i^{(\alpha)}(u) = (1 - u)P_{i,q}^{(\alpha)} + uP_{i,q+1}^{(\alpha)},
$$

(48)
with \( u = \frac{L}{T_{\text{min}}} (t - t_k - \theta) \), \( V_2(x_i, 1, t) \), \( \theta(t_{k+1}, t, x_i) \) are same with \( (20) \) and \( (21) \).

Now, we have the following theorem based on time-scheduled Lyapunov functional \( (45) \).

**Theorem 3.** If the SMJS \((1)\) is mean-square asymptotically stable if there exist positive-definite matrices \( P_{i,q}^{(a)}, Q^{(a)}, Z^{(a)}, S^{(a)}, R^{(a)} \), any matrices \( U_i \) with appropriate dimensions such that the following LMIs hold for \( q = 0, 1, \ldots, L, i \in \mathcal{N}, a = 1, 2, \ldots, r \):

\[
\begin{bmatrix}
\phi_{i,q}^{(a)} \left( H_i^{(a)} \right)^T P_{i,q}^{(a)} h \left( H_i^{(a)} \right)^T Z^{(a)} \left( H_i^{(a)} \right)^T S^{(a)} \\
* -P_{i,q}^{(a)} & 0 & 0 \\
* * -hZ^{(a)} & 0 \\
* * * -S^{(a)}
\end{bmatrix} < 0, \ q = 0, 1, \ldots, L - 1. \tag{49}
\]

\[
\begin{bmatrix}
\phi_{i, q+1}^{(a)} \left( H_i^{(a)} \right)^T P_{i, q+1}^{(a)} h \left( H_i^{(a)} \right)^T S^{(a)} \left( H_i^{(a)} \right)^T S^{(a)} \\
* -P_{i, q+1}^{(a)} & 0 & 0 \\
* * -hZ^{(a)} & 0 \\
* * * -S^{(a)}
\end{bmatrix} < 0, \ q = 0, 1, \ldots, L - 1. \tag{50}
\]

\[
\begin{bmatrix}
\bar{\phi}_{i, L}^{(a)} \left( H_i^{(a)} \right)^T P_{i, L}^{(a)} h \left( H_i^{(a)} \right)^T Z^{(a)} \left( H_i^{(a)} \right)^T S^{(a)} \\
* -P_{i, L}^{(a)} & 0 & 0 \\
* * -hZ^{(a)} & 0 \\
* * * -S^{(a)}
\end{bmatrix} < 0. \tag{51}
\]

\[
P_{j,0}^{(a)} - P_{i,l}^{(a)} < 0, \ i \neq j. \tag{52}
\]

where

\[
T_k \subset \{ T_{\text{min}}, T_{\text{max}} \},
\]

\[
\phi_{i,q}^{(a)} = \eta_{1,i,q}^{(a)} + \eta_2^{(a)} + \eta_3^{(a)} + \eta_4^{(a)} + \eta_5^{(a)} + \eta_6^{(a)},
\]

\[
\phi_{i, q+1}^{(a)} = \tilde{\eta}_{1,i,q+1}^{(a)} + \eta_2^{(a)} + \eta_3^{(a)} + \eta_4^{(a)} + \eta_5^{(a)} + \eta_6^{(a)},
\]

\[
\phi_{i,L}^{(a)} = \tilde{\eta}_{1,i,L}^{(a)} + \eta_2^{(a)} + \eta_3^{(a)} + \eta_4^{(a)} + \eta_5^{(a)} + \eta_6^{(a)},
\]

\[
\eta_{1,i,q}^{(a)} = \text{He} \left\{ e_1^T P_{i,q}^{(a)} A_i^{(a)} e_1 + e_1^T P_{i,q}^{(a)} A_{di}^{(a)} e_2 - e_1^T P_{i,q}^{(a)} B_i^{(a)}(t) e_3 \right\}
\]

\[
+ e_1^T \left( \sum_{j=1}^{s} \pi_{ij} P_{j,q}^{(a)} + \frac{L}{T_{\text{min}}} \left( P_{i,q}^{(a)} - P_{i,q}^{(a)} \right) \right) e_1,
\]

\[
\tilde{\eta}_{1,i,q+1}^{(a)} = \text{He} \left\{ e_1^T P_{i,q+1}^{(a)} A_i^{(a)} e_1 + e_1^T P_{i,q+1}^{(a)} A_{di}^{(a)} e_2 - e_1^T P_{i,q+1}^{(a)} B_i^{(a)}(t) e_3 \right\}
\]

\[
+ e_1^T \left( \sum_{j=1}^{s} \pi_{ij} P_{j,q+1}^{(a)} + \frac{L}{T_{\text{min}}} \left( P_{i,q+1}^{(a)} - P_{i,q}^{(a)} \right) \right) e_1,
\]

\[
\tilde{\eta}_{1,i,L}^{(a)} = \text{He} \left\{ e_1^T P_{i,L}^{(a)} A_i^{(a)} e_1 + e_1^T P_{i,L}^{(a)} A_{di}^{(a)} e_2 - e_1^T P_{i,L}^{(a)} B_i^{(a)}(t) e_3 \right\} + e_1^T \left( \sum_{j=1}^{s} \pi_{ij} P_{j,L}^{(a)} \right) e_1.
\]

\( \eta_2^{(a)}, \eta_3^{(a)}, \eta_4^{(a)}, \eta_5^{(a)} \) and \( \eta_{1,i,q}^{(a)} \) have already been given in Theorem 1.
Proof. Let $\mathcal{L}$ be the week infinitesimal operator of the random process $\{x_i, r(t), t \geq 0\}$ along system (1) and we have

$$
\mathcal{L}\hat{V}_i(x_i, i, t) = \sum_{a=1}^{m} \theta_a \left\{ (1 - u) \left[ 2x^T(t)P_{i,q}(t) + \text{Tr} \left\{ g_i^T(t)P_{i,q}(t) g_i(t) \right\} \right]
+ x^T(t) \left( \sum_{j=1}^{s} \pi_{ij} P_{j,q}(t) \right) x(t) + x^T(t) \left( \frac{L}{T_{\min}} \left( P_{i,q+1} - P_{i,q} \right) \right) x(t) \right\}
+ u \left[ 2x^T(t)P_{i,q+1}(t) + \text{Tr} \left\{ g_i^T(t)P_{i,q+1}(t) g_i(t) \right\} \right]
+ x^T(t) \left( \sum_{j=1}^{s} \pi_{ij} P_{j,q+1}(t) \right) x(t) + x^T(t) \left( \frac{L}{T_{\min}} \left( P_{i,q+1} - P_{i,q} \right) \right) x(t) \right\}
= \sum_{a=1}^{m} \theta_a \left\{ (1 - u) \left[ \eta_{li,q} + \text{Tr} \left\{ g_j^T(t)P_{i,q}(t) g_j(t) \right\} \right]
+ u \left[ \eta_{li,q} + \text{Tr} \left\{ g_j^T(t)P_{i,q+1}(t) g_j(t) \right\} \right] \right\}.
$$

(53)

Then, from (23)-(29) and (53), we have

$$
\mathbb{E} (\mathcal{L}W(t)) = \mathbb{E} \left( \sum_{a=1}^{m} \theta_a \left[ (1 - u)\Phi_{i,q}^{(a)} + u\hat{\Phi}_{i,q+1}^{(a)} \right] \xi_i(t) \right).
$$

(54)

where

$$
\Phi_{i,q}^{(a)} = \Phi_{i,q}^{(a)} + \left( H_{i}^{(a)} \right)^T P_{i,q} H_{i}^{(a)} + h \left( H_{i}^{(a)} \right)^T Z(a)^T S(a) H_{i}^{(a)},
\hat{\Phi}_{i,q+1}^{(a)} = \hat{\Phi}_{i,q+1}^{(a)} + \left( H_{i}^{(a)} \right)^T P_{i,q+1} H_{i}^{(a)} + h \left( H_{i}^{(a)} \right)^T Z(a)^T S(a) H_{i}^{(a)}.
$$

When $\Phi_{i,q}^{(a)} < 0, \hat{\Phi}_{i,q+1}^{(a)} < 0$, by Schur’s complement, LMIs conditions (49)-(50) are obtained.

When $t \in \lambda_{k,L}$. $P_{i,i}(t) = P_{i,i}(t)$ following the similar line as Theorem 1, LMIs condition (51) is obtained. Moreover, at the discrete-time sampling instants $t_k$, as explained in the work of Shaked and Gershon, the Lyapunov functional is nonincreasing. Therefore, (52) is guaranteed. This completes the proof. 

Now, we can get the following corollary to guarantee the mean-square asymptotical stability of nominal continuous-time Itô SMJs (32) based on Theorem 3.

**Corollary 3.** Itô SMJS (32) is mean-square asymptotically stable if there exist positive-definite matrices $P_{i,q}, Q, Z, S, R$ with appropriate dimensions such that the following LMIs hold for $q = 0, 1, ...L, i \in \mathcal{N}$:

$$
\begin{bmatrix}
\phi_{i,q} & H_{i}^{T} P_{i,q} & h H_{i}^{T} Z & H_{i}^{T} S \\
\ast & -P_{i,q} & 0 & 0 \\
\ast & \ast & -hZ & 0 \\
\ast & \ast & \ast & -S
\end{bmatrix}
< 0, q = 0, 1, \ldots, L - 1.
$$

(55)

$$
\begin{bmatrix}
\phi_{i,q+1} & H_{i}^{T} P_{i,q+1} & h H_{i}^{T} S & H_{i}^{T} S \\
\ast & -P_{i,q+1} & 0 & 0 \\
\ast & \ast & -hZ & 0 \\
\ast & \ast & \ast & -S
\end{bmatrix}
< 0, q = 0, 1, \ldots, L - 1.
$$

(56)

$$
\begin{bmatrix}
\phi_{i,i} & H_{i}^{T} P_{i,i} & h H_{i}^{T} Z & H_{i}^{T} S \\
\ast & -P_{i,i} & 0 & 0 \\
\ast & \ast & -hZ & 0 \\
\ast & \ast & \ast & -S
\end{bmatrix}
< 0.
$$

(57)

$$
P_{j,0} - P_{i,i} < 0, \ i \neq j.
$$

(58)
where
\[ T_k \in \{ T_{\text{min}}, T_{\text{max}} \}, \]
\[ \phi_{i,q} = \eta_{1i,q} + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_{\delta i}, \]
\[ \dot{\phi}_{i,q+1} = \dot{\eta}_{1i,q+1} + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_{\delta i}, \]
\[ \phi_{i,L} = \dot{\eta}_{1i,L} + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_{\delta i}. \]
\[ \eta_{1i,q} = \text{He} \left\{ e^{T}P_{i,q}A_i e_1 + e^{T}P_{i,q}A_{di} e_2 - e^{T}P_{i,q}B_i K_i(t) e_3 \right\} + e_{i}^{T} \left( \sum_{j=1}^{s} \pi_{ij} P_{j,q} + \frac{L}{T_{\text{min}}} (P_{i,q+1} - \dot{P}_{i,q}) \right) e_1, \]
\[ \eta_{1i,q+1} = \text{He} \left\{ e^{T}P_{i,q+1}A_i e_1 + e^{T}P_{i,q+1}A_{di} e_2 - e^{T}P_{i,q+1}B_i K_i(t) e_3 \right\} + e_{i}^{T} \left( \sum_{j=1}^{s} \pi_{ij} P_{j,q+1} + \frac{L}{T_{\text{min}}} (P_{i,q+1} - \dot{P}_{i,q}) \right) e_1, \]
\[ \dot{\eta}_{1i,L} = \text{He} \left\{ e^{T}P_{i,L}A_i e_1 + e^{T}P_{i,L}A_{di} e_2 - e^{T}P_{i,L}B_i K_i(t) e_3 \right\} + e_{i}^{T} \left( \sum_{j=1}^{s} \pi_{ij} P_{j,L} \right) e_1. \]
\[ \eta_2, \eta_3, \eta_4, \eta_5, \eta_{\delta i} \text{ have been given in Corollary 1.} \]

**Proof.** Construct a Lyapunov-Krasovskii functional candidate
\[ \mathcal{W}(x_i, i, t) = V_1(x_i, i, t) + V_2(x_i, i, t) + \tilde{\theta}(t_{k+1}, t, x_i), \] (59)

where
\[ V_1(x_i, i, t) = x^{T}(t) P_i(t) x(t), \] (60)

with
\[ P_i(t) = \begin{cases} P_i(u), & t \in \lambda_{k,q}, q = 0, \ldots, L - 1. \\ P_{i,L}, & t \in \lambda_{k,L}, q = L, \end{cases} \] (61)

where
\[ P_i(u) = (1 - u)P_{i,q} + uP_{i,q+1}, \] (62)

with \[ u = \frac{L}{T_{\text{min}}} (t - t_k - \theta_q), \]
\[ V_2(x_i, i, t), \tilde{\theta}(t_{k+1}, t, x_i) \text{ are same with (39) and (40).} \]

Then, following the similar line as in the proof of Theorem 3, the proof can be completed. \(\square\)

### 4.2 Time-dependent robust sampled-data controller design

**Theorem 4.** For given positive constants \( \lambda_i \), Itô SMJS (1) is mean-square asymptotically stable if there exist positive-definite matrices \( Q_{i,q}^{(a)}, Z_{i,q}^{(a)}, S_{i,q}^{(a)}, R_{i,q}^{(a)}, \dot{Q}_{i,q}^{(a)}, \dot{Z}_{i,q}^{(a)}, \dot{S}_{i,q}^{(a)}, Q_{i,q+1}^{(a)}, R_{i,q+1}^{(a)}, Z_{i,q+1}^{(a)}, S_{i,q+1}^{(a)} \) with appropriate dimensions such that the following LMIs hold for \( q = 0, 1, \ldots, L, i \in N, \alpha = 1, 2, \ldots, r \):

\[ \begin{bmatrix} Q_{i,q}^{(a)} & \dot{Q}_{i,q}^{(a)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{H}_{i,q}^{(a)} & 0 & 0 & 0 & \end{bmatrix} + \begin{bmatrix} \dot{H}_{i,q+1}^{(a)} & 0 & 0 & 0 & \end{bmatrix} \begin{bmatrix} Q_{i,q+1}^{(a)} & \dot{Q}_{i,q+1}^{(a)} & 0 & 0 & 0 \end{bmatrix} \leq 0, \] (63)

\[ q = 0, 1, \ldots, L - 1. \]

\[ \begin{bmatrix} \dot{Q}_{i,q+1}^{(a)} & 0 & 0 & 0 & \end{bmatrix} \begin{bmatrix} \dot{H}_{i,q+1}^{(a)} & 0 & 0 & 0 & \end{bmatrix} \begin{bmatrix} \dot{Q}_{i,q+1}^{(a)} & 0 & 0 & 0 & \end{bmatrix} \begin{bmatrix} \dot{H}_{i,q+1}^{(a)} & 0 & 0 & 0 & \end{bmatrix} \begin{bmatrix} e^{T}A_{i,q+1}^{(a)} & 0 \end{bmatrix} \leq 0, \] (64)

\[ q = 0, 1, \ldots, L - 1. \]
\[
\begin{bmatrix}
\dot{\varphi}^{(a)}_{L_L} (Q^{(a)}_{L_L})^T \\
\vdots \\
\dot{\varphi}^{(a)}_{L_L} (Q^{(a)}_{L_L})^T \\
\end{bmatrix} + h(Q^{(a)}_{L_L}) (H_i^{(a)})^T (Q^{(a)}_{L_L})^T (H_i^{(a)})^T e_i^T \Lambda^{(a)}_{L_L} < 0,
\]

where

\[
T_k \in \{T_{\text{min}}, T_{\text{max}}\},
\]

\[
\varphi^{(a)}_{L_L} = \varphi^{(a)}_{L_L} + \varphi^{(a)}_{L_L} + \varphi^{(a)}_{L_L} + \varphi^{(a)}_{L_L} + \varphi^{(a)}_{L_L},
\]

\[
\dot{\varphi}^{(a)}_{L_L} = \dot{\varphi}^{(a)}_{L_L} + \dot{\varphi}^{(a)}_{L_L} + \dot{\varphi}^{(a)}_{L_L} + \dot{\varphi}^{(a)}_{L_L} + \dot{\varphi}^{(a)}_{L_L} + \dot{\varphi}^{(a)}_{L_L},
\]

\[
Q^{(a)}_{L_L} - Q^{(a)}_{L_L} < 0, i \neq j.
\]
Then, according to the proof of Theorem 3, we can get

\[ Q_{ij,q} = \sum_{j=1,\neq j}^{s} \lambda_{ij} Q_{il,q}^{(a)} Q_{l,j,q}^{(a)}^{-1} = \Lambda_{1l,q} \left( \Lambda_{2l,q} \right)^{-1} \Lambda_{1l,q}^T, \]

\[ \Lambda_{1l,q} = \left[ \sqrt{\alpha_{il} Q_{il,q}}, \ldots, \sqrt{\alpha_{i-1,l} Q_{i-1,l,q}}, \sqrt{\alpha_{i+1,l} Q_{i+1,l,q}}, \ldots, \sqrt{\alpha_{il} Q_{il,q}} \right], \]

\[ \Lambda_{2l,q} = \text{diag} \left\{ Q_{i,q}^{(a)}, \ldots, Q_{i-1,q}^{(a)}, Q_{i+1,q}^{(a)}, \ldots, Q_{s,q}^{(a)} \right\}, \]

\[ Q_{ij,q+1}^{(a)} = \sum_{j=1,\neq j}^{s} \lambda_{ij} Q_{il,q+1}^{(a)} \left( Q_{ij,q+1}^{(a)} \right)^{-1} = \Lambda_{1l,q+1} \left( \Lambda_{2l,q+1} \right)^{-1} \Lambda_{1l,q+1}^T, \]

\[ \Lambda_{1l,q+1} = \left[ \sqrt{\alpha_{il} Q_{il,q+1}}, \ldots, \sqrt{\alpha_{i-1,l} Q_{i-1,l,q+1}}, \sqrt{\alpha_{i+1,l} Q_{i+1,l,q+1}}, \ldots, \sqrt{\alpha_{il} Q_{il,q+1}} \right], \]

\[ \Lambda_{2l,q+1} = \text{diag} \left\{ Q_{1,q+1}^{(a)}, \ldots, Q_{i-1,q+1}^{(a)}, Q_{i+1,q+1}^{(a)}, \ldots, Q_{s,q+1}^{(a)} \right\}, \]

\[ Q_{ij,L}^{(a)} = \sum_{j=1,\neq j}^{s} \lambda_{ij} Q_{il,L}^{(a)} \left( Q_{ij,L}^{(a)} \right)^{-1} = \Lambda_{1l,L} \left( \Lambda_{2l,L} \right)^{-1} \Lambda_{1l,L}^T, \]

\[ \Lambda_{1l,L} = \left[ \sqrt{\alpha_{il} Q_{il,L}}, \ldots, \sqrt{\alpha_{i-1,l} Q_{i-1,l,L}}, \sqrt{\alpha_{i+1,l} Q_{i+1,l,L}}, \ldots, \sqrt{\alpha_{il} Q_{il,L}} \right], \]

\[ \Lambda_{2l,L} = \text{diag} \left\{ Q_{1,L}^{(a)}, \ldots, Q_{i-1,L}^{(a)}, Q_{i+1,L}^{(a)}, \ldots, Q_{s,L}^{(a)} \right\}. \]

The time-dependent controller can be obtained by

\[
K_{i}^{(a)}(t) = \begin{cases} 
\hat{K}_{i}^{(a)}(t) & \text{if } t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1, \\
K_{i}^{(a)}(t) & \text{if } t \in \lambda_{k,L}, 
\end{cases}
\]

where

\[ K_{i}^{(a)}(t) = (1 - u) \hat{K}_{i}^{(a)} + u \hat{K}_{i}^{(a+1)}, \]

with \( u = \frac{L}{T_{\text{min}}} (t - t_k - \theta_q). \)

**Proof.** Constructing a Lyapunov-Krasovskii functional candidate

\[ W(x, i, t) = V_1(x, i, t) + V_2(x, i, t) + \theta(t_k+1, t, x), \]

where

\[
V_1(x, i, t) = \sum_{a=1}^{m} \theta_a x^T(t) \left( Q_i^{(a)}(t) \right)^{-1} x(t),
\]

with

\[ Q_i^{(a)}(t) = \begin{cases} 
Q_i^{(a)}(u), t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1, \\
Q_i^{(a)}(u), t \in \lambda_{k,L}, 
\end{cases}
\]

where

\[ Q_i^{(a)}(u) = (1 - u) Q_i^{(a)} + u Q_i^{(a+1)}, \]

with \( u = \frac{L}{T_{\text{min}}} (t - t_k - \theta_q). \), \( V_2(x, i, t) \), \( \theta(t_k+1, t, x) \) are same with (20) and (21).

Let \( L \) be the weak infinitesimal operator of the random process \( \{x, r(t), t \geq 0\} \) along system (1) and we have

\[
L V_1(x, i, t) = \sum_{a=1}^{m} \theta_a \left\{ 2 x^T(t) \left( Q_i^{-1}(t) \right)^{(a)} f_i(t) + \text{Tr} \left\{ \left( g_i^T(t) \left( Q_i^{-1}(t) \right)^{(a)} \right)^T g_i(t) \right\} \right. \\
+ x^T(t) \left( \sum_{j=1}^{s} \lambda_{ij} x^T \left( Q_j^{-1}(t) \right)^{(a)} \right) x(t) - x^T(t) \left( \left( Q_i^{-1}(t) \right)^{(a)} \right) x(t) \right\}. \]

Then, according to the proof of Theorem 3, we can get

\[
\left[ \begin{array}{ccc}
\phi_i^{(a)} & (H_i^{(a)})^T & Z^{(a)} \\
* & -(Q_i^{-1}(t))^{(a)} & 0 \\
* & * & -h S^{(a)}
\end{array} \right] < 0,
\]

(73)
where
\[
\phi_i^{(a)} = \eta_{i1}^{(a)} + \eta_2^{(a)} + \eta_3^{(a)} + \eta_4^{(a)} + \eta_5^{(a)} + \eta_{6i}^{(a)},
\]
\[
\eta_i^{(a)} = \text{He} \left\{ e_t^\top (Q_i^{-1}(t))^{(a)} A_i^{(a)} e_1 + e_t^\top (Q_i^{-1}(t))^{(a)} A_{di}^{(a)} e_2 - e_t^\top (Q_i^{-1}(t))^{(a)} B_i^{(a)} \hat{\kappa}_i(t) e_3 \right\}
\]
\[
+ e_t^\top \left( \sum_{j=1}^2 \pi_{ij} (Q_j^{-1}(t))^{(a)} - (Q_i^{-1}(t))^{(a)} (Q_i^{-1}(t))^{(a)} \right) e_1.
\]

\(\eta_2^{(a)}, \eta_3^{(a)}, \eta_4^{(a)}, \eta_5^{(a)}, \) and \(\eta_{6i}^{(a)}\) are same with those in Theorem 3.

Define the following new variables:
\[
\dot{Q}_{i,q}^{(a)} = Q_{i,q}^{(a)} Q_{i,q}^{(a)} \dot{Q}_{i,q}^{(a)} = Q_{i,q}^{(a)} Q_{i,q}^{(a)} + \dot{Q}_{i,q}^{(a)},
\]
\[
Z_{i,q}^{(a)} = Q_{i,q}^{(a)} Z_{i,q}^{(a)} Q_{i,q}^{(a)},
\]
\[
S_{i,q}^{(a)} = Q_{i,q}^{(a)} S_{i,q}^{(a)} Q_{i,q}^{(a)} + \dot{S}_{i,q}^{(a)},
\]
\[
R_{i,q}^{(a)} = Q_{i,q}^{(a)} R_{i,q}^{(a)} Q_{i,q}^{(a)} + \dot{R}_{i,q}^{(a)},
\]
\[
Q_{i,q}^{(a)} = Q_{i,q}^{(a)} Q_{i,q}^{(a)} + \dot{Q}_{i,q}^{(a)},
\]
\[
S_{i,q}^{(a)} = Q_{i,q}^{(a)} S_{i,q}^{(a)} Q_{i,q}^{(a)} + \dot{S}_{i,q}^{(a)},
\]
\[
U_i = \lambda_i \left( Q_{i,q}^{(a)} \right)^{-1}.
\]

Then, multiply
\[
\text{diag} \left\{ Q_i^{(a)}(t), Q_i^{(a)}(t), Q_i^{(a)}(t), Q_i^{(a)}(t), Q_i^{(a)}(t), (Z^{(a)})^{-1}, (S^{(a)})^{-1} \right\}
\]
in both sides of LMIs (73).

Then, based on the aforementioned new variables and following the consistent interpretation of Theorem 3, LMI conditions (63)-(66) can be obtained.

Therefore, we can obtain the following time-dependent controller (67) to guarantee the mean-square asymptotical stability of system (32).

From Theorem 3, we can get the time-dependent sampled-data controller for nominal Itô SMJSs (32). It is summarized as Corollary 4.

**Corollary 4.** Given positive constants \( \lambda_i \). Itô SMJS (32) is mean-square asymptotically stable if there exist positive-definite matrices \( Q_{i,q}, Z_i, S_{i,q}, R_{i,q}, \hat{Q}_{i,q}, Z, S, Q_{i,q}, R_{i,q}, Z_{i,q}, S_{i,q} \) with appropriate dimensions such that the following LMIs hold for \( q = 0, 1, \ldots, L, i \in N' \):

\[
\begin{bmatrix}
\varphi_{i,q} & Q_{i,q} H_t^\top & h Q_{i,q} H_t^\top & Q_{i,q} H_t^\top & e_t^\top \Lambda_i, Q \n * & -Q_{i,q} & 0 & 0 & 0 
* & * & -h Z^{-1} & 0 & 0 
* & * & * & -S^{-1} & 0
\end{bmatrix} < 0
\]

\( q = 0, 1, \ldots, L - 1. \)  \hspace{1cm} (74)

\[
\begin{bmatrix}
\varphi_{i,q+1} & Q_{i,q+1} H_t^\top & h Q_{i,q+1} H_t^\top & Q_{i,q+1} H_t^\top & e_t^\top \Lambda_{i,q+1} 
* & -Q_{i,q+1} & 0 & 0 & 0 
* & * & -h Z^{-1} & 0 & 0 
* & * & * & -S^{-1} & 0 
* & * & * & * & -\Lambda_{2i,q+1}
\end{bmatrix} < 0
\]

\( q = 0, 1, \ldots, L - 1. \)  \hspace{1cm} (75)

\[
\begin{bmatrix}
\varphi_{i,l} & Q_{i,l} H_t^\top & h Q_{i,l} H_t^\top & Q_{i,l} H_t^\top & e_t^\top \Lambda_{i,l} 
* & -Q_{i,l} & 0 & 0 & 0 
* & * & -h Z^{-1} & 0 & 0 
* & * & * & -S^{-1} & 0 
* & * & * & * & -\Lambda_{2i,l}
\end{bmatrix} < 0
\]

\( Q_{i,l} - Q_{j,0} < 0, \ i \neq j, \) \hspace{1cm} (76)

\[
Q_{i,l} - Q_{j,0} < 0, \ i \neq j, \) \hspace{1cm} (77)
where
\[
T_k \in \{T_{\text{min}}, T_{\text{max}}\},
\]
\[
\varphi_{ij} = \chi_{ij} + \chi_{2ij} + \chi_{3ij} + \chi_{4ij} + \chi_{5ij} + \chi_{6ij},
\]
\[
\Phi_{ij+1} = \hat{\chi}_{ij+1} + \hat{\chi}_{2ij+1} + \hat{\chi}_{3ij+1} + \hat{\chi}_{4ij+1} + \hat{\chi}_{5ij+1} + \hat{\chi}_{6ij+1},
\]
\[
\Phi_{iL} = \hat{\chi}_{iL} + \hat{\chi}_{3iL} + \hat{\chi}_{4iL} + \hat{\chi}_{5iL} + \hat{\chi}_{6iL},
\]
\[
\zeta_{ij} = \text{He} \left\{ e_i^T A_i Q_{ij} e_1 + e_i^T A_{di} Q_{ij} e_2 - e_i^T B_i K_{ij} e_3 \right\} + e_i^T \left( -\frac{L}{T_{\text{min}}} (Q_{ij+1} - Q_{ij}) + Q_{ij+1} + \pi_i Q_{ij} \right) e_1,
\]
\[
\hat{\zeta}_{ij+1} = \text{He} \left\{ e_i^T A_i Q_{ij+1} e_1 + e_i^T A_{di} Q_{ij+1} e_2 - e_i^T B_i K_{ij+1} e_3 \right\}
+ e_i^T \left( -\frac{L}{T_{\text{min}}} (Q_{ij+1} - Q_{ij}) + Q_{ij+1} + \pi_i Q_{ij} \right) e_1,
\]
\[
\zeta_{iL} = \text{He} \left\{ e_i^T A_i Q_{iL} e_1 + e_i^T A_{di} Q_{iL} e_2 - e_i^T B_i K_{iL} e_3 \right\} + e_i^T \left( Q_{iL+1} + \pi_i Q_{iL} \right) e_1,
\]
\[
\zeta_{2iL} = e_i^T \left( (1 - u) \hat{Q}_{iL} + 2u Q_{iL} \right) e_1 - e_i^T \left( (1 - u) \hat{Q}_{iL} + 2u Q_{iL} \right) e_2,
\]
\[
\hat{\xi}_{2iL+1} = e_i^T u \hat{Q}_{iL+1} e_1 - e_i^T u \hat{Q}_{iL+1} e_2,
\]
\[
\hat{\zeta}_{3iL} = \left( (1 - u) S_{iL} + 2u S_{iL} \right) (e_1 - e_3) + \text{He} \left\{ T_k \left( e_i^T - e_i^T \right) \right\} \left( (1 - u) S_{iL} + 2u S_{iL} \right) e_6
+ T_k e_i^T \left( (1 - u) R_{iL} + 2u R_{iL} \right) e_6,
\]
\[
\hat{\zeta}_{4iL} = \left( e_i^T - e_i^T \right) S_{iL} (e_1 - e_3) + \text{He} \left\{ T_k \left( e_i^T - e_i^T \right) \right\} \left( S_{iL} e_6 + T_k e_i^T \right) R_{iL} e_6,
\]
\[
\zeta_{6iL} = \left( e_i^T - e_i^T \right) (1 - u) Z_{iL} + 2u Z_{iL} (e_1 - e_4) + e_i^T \left( (1 - u) Z_{iL} + 2u Z_{iL} \right) e_2,
\]
\[
\hat{\zeta}_{6L+1} = e_i^T \left( e_i^T - e_i^T \right) Z_{iL+1} (e_1 - e_4) + e_i^T \left( e_i^T - e_i^T \right) Z_{iL+1} e_2,
\]
\[
\zeta_{5L+1} = \frac{1}{T_{\text{max}}} e_i^T \left( (1 - u) R_{iL} + 2u R_{iL} \right) e_5,
\]
\[
\hat{\zeta}_{5L+1} = \frac{1}{T_{\text{max}}} e_i^T \left( (1 - u) R_{iL} + 2u R_{iL} \right) e_5,
\]
\[
\hat{\xi}_{5iL} = \text{He} \left\{ e_i^T \lambda_i Q_{iL} e_6 + e_i^T A_i Q_{iL} e_6 - e_i^T \lambda_i \hat{K}_{iL} B_i e_6 - \lambda_i e_i^T (Q_{iL} + Q_{iL}) e_6 \right\},
\]
\[
\hat{\xi}_{6iL+1} = \text{He} \left\{ e_i^T \lambda_i Q_{iL+1} e_6 + e_i^T A_i Q_{iL+1} e_6 - e_i^T \lambda_i \hat{K}_{iL+1} B_i e_6 - \lambda_i e_i^T (Q_{iL+1} + Q_{iL}) e_6 \right\},
\]
\[
\hat{\xi}_{6iL+1} = \text{He} \left\{ e_i^T \lambda_i Q_{iL+1} e_6 + e_i^T A_i Q_{iL+1} e_6 - e_i^T \lambda_i \hat{K}_{iL+1} B_i e_6 - \lambda_i e_i^T (Q_{iL+1} + Q_{iL}) e_6 \right\},
\]
\[
Q_{ij} = \sum_{j=1}^{s} \pi_{ij} Q_{ij} (Q_{ij})^{-1} Q_{ij} = \lambda_{1ij} \lambda_{2ij}^{-1} (\lambda_{1ij})^T,
\]
\[
\lambda_{1ij} = \left[ \sqrt{\pi_{1j} Q_{ij}}, \ldots, \sqrt{\pi_{1j-1} Q_{ij}}, \sqrt{\pi_{1j+1} Q_{ij}}, \ldots, \sqrt{\pi_{1j} Q_{ij}} \right],
\]
\[
\lambda_{2iq+1} = \text{diag} \left\{ Q_{ij+1}, Q_{ij+1}, Q_{ij+1}, \ldots, Q_{iq+1} \right\},
\]
\[
Q_{ij+1} = \sum_{j=1}^{s} \pi_{ij} Q_{ij+1} (Q_{ij+1})^{-1} Q_{ij+1} = \lambda_{1iq+1} \lambda_{2iq+1}^{-1} (\lambda_{1iq+1})^T,
\]
\[
\lambda_{1iq+1} = \left[ \sqrt{\pi_{1i} Q_{iq+1}}, \ldots, \sqrt{\pi_{1i-1} Q_{iq+1}}, \sqrt{\pi_{1i+1} Q_{iq+1}}, \ldots, \sqrt{\pi_{1i} Q_{iq+1}} \right],
\]
\[
\lambda_{2iq+1} = \text{diag} \left\{ Q_{iq+1}, Q_{iq+1}, Q_{iq+1}, \ldots, Q_{iq+1} \right\},
\]
\[
\lambda_{1il} = \left[ \sqrt{\pi_{1i} Q_{il}}, \ldots, \sqrt{\pi_{1i-1} Q_{il}}, \sqrt{\pi_{1i+1} Q_{il}}, \ldots, \sqrt{\pi_{1i} Q_{il}} \right],
\]
\[
\lambda_{2il} = \text{diag} \left\{ Q_{il}, Q_{il}, Q_{il}, \ldots, Q_{il} \right\}.
\]
The time-dependent sampled-data controller can be obtained by

\[
K_i(t) = \begin{cases} 
\hat{K}_i(t)(1 - u)Q_{i,q} + uQ_{i,q+1}^{-1}, \\
\hat{K}_i(L)Q_{i,L}^{-1}, 
\end{cases} 
\]

\[t \in \lambda_{k,q}, q = 0, 1, \ldots, L - 1.\]

(78)

where

\[
\hat{K}_i(t) = (1 - u)\hat{K}_{i,q} + u\hat{K}_{i,q+1},
\]

with \(u = \frac{L}{t_{min}}(t - t_k - \theta_q)\).

5 | NUMERICAL EXAMPLES

Example. Consider a two-vertex polytopic Itô SMJs (1) with two modes and the following parameters.

Mode 1:

\[
A_1^{(1)} = \begin{bmatrix} 0.5 & -0.1 \\ 3.5 & -0.1 \end{bmatrix}, A_1^{(2)} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, B_1^{(1)} = \begin{bmatrix} 1 & 0 \\ -0.2 & 0.8 \end{bmatrix},
\]

\[
A_1^{(2)} = \begin{bmatrix} -1 & 0.1 \\ 2 & -0.1 \end{bmatrix}, A_1^{(2)} = \begin{bmatrix} -0.1 & 0.8 \\ 0.3 & 0.1 \end{bmatrix}, B_1^{(2)} = \begin{bmatrix} 1 & 0.3 \\ -0.6 & 0.8 \end{bmatrix},
\]

\[
E_1^{(1)} = \begin{bmatrix} -1.1 & 0.4 \\ 0.2 & -0.3 \end{bmatrix}, E_1^{(2)} = \begin{bmatrix} 0.09 & -0.6 \\ 0.3 & 0.02 \end{bmatrix},
\]

\[
E_1^{(2)} = \begin{bmatrix} 1.1 & 0.5 \\ 0.2 & 0.3 \end{bmatrix}, E_1^{(2)} = \begin{bmatrix} 0.08 & 0.7 \\ 0.5 & 0.02 \end{bmatrix}.
\]

Mode 2:

\[
A_2^{(1)} = \begin{bmatrix} 0.4 & -0.5 \\ 0.3 & -0.15 \end{bmatrix}, A_2^{(2)} = \begin{bmatrix} 0.1 & 0.3 \\ 0.3 & 0.1 \end{bmatrix}, B_2^{(1)} = \begin{bmatrix} 1.1 & 0 \\ -0.2 & 1 \end{bmatrix},
\]

\[
A_2^{(2)} = \begin{bmatrix} 0.7 & 0.5 \\ 0.3 & -0.15 \end{bmatrix}, A_2^{(2)} = \begin{bmatrix} 0.1 & 0.3 \\ 0.3 & 0.1 \end{bmatrix}, B_2^{(2)} = \begin{bmatrix} 1.1 & 0.3 \\ -0.2 & 1 \end{bmatrix},
\]

\[
E_2^{(1)} = \begin{bmatrix} -0.1 & -0.4 \\ 0.4 & 0.4 \end{bmatrix}, E_2^{(2)} = \begin{bmatrix} 0.2 & 0.5 \\ 0.7 & 0.3 \end{bmatrix},
\]

\[
E_2^{(2)} = \begin{bmatrix} 0.5 & -0.4 \\ 0.4 & -0.2 \end{bmatrix}, E_2^{(2)} = \begin{bmatrix} 0.3 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}.
\]

Let \(T_{\min} = 0.01, T_{\max} = 0.03, \lambda_1 = 0.1, h = 0.1, \Pi = \begin{bmatrix} -2 & 2 \\ 1.5 & -1.5 \end{bmatrix}\), by solving LMIs in Theorem 2, we can get

\[
K_1^{(1)} = \begin{bmatrix} 6.8265 & 1.0985 \\ 4.2941 & 5.5819 \end{bmatrix}, K_2^{(2)} = \begin{bmatrix} 3.5548 & 0.5466 \\ 6.3650 & 6.9158 \end{bmatrix},
\]

\[
K_2^{(1)} = \begin{bmatrix} 4.8189 & 0.4861 \\ 1.6734 & 3.8755 \end{bmatrix}, K_2^{(2)} = \begin{bmatrix} 5.2391 & -0.1482 \\ 2.1414 & 4.2143 \end{bmatrix}.
\]

For system \(dx(t) = [A_1^{(1)} x(t) + A_1^{(2)} x(t - h) + B_1^{(1)} u(t)]dt + [E_1^{(1)} x(t) + E_1^{(2)} x(t - h)]dw(t), i = 1, 2\), the state response of the open-loop system and Markovian jump signal are presented in Figure 1. By the state feedback stabilization of time-independent sampled-data controller \(K_i^{(1)}, i = 1, 2\), the closed-loop state responses and Markovian jump signal are shown in Figure 2. Therefore, we can verify that the method of designing time-independent state feedback sampled-data controller is effective to stabilize system (1). The corresponding aperiodic sampling control input and aperiodic sampling intervals are shown in Figure 3.

Furthermore, suppose \(L = 2, T_{\min} = 0.01, T_{\max} = 0.06, \theta_k = 0.01\). When \(q = 0\), let \(t = 0.011\). When \(q = 1\), let \(t = 0.016\). The obtained sampling controllers with different cases by solving the LMIs in Theorem 4 are listed in Table 1. For system \(dx(t) = [A_1^{(1)} x(t) + A_1^{(2)} x(t - h) + B_1^{(1)} u(t)]dt + [E_1^{(1)} x(t) + E_1^{(2)} x(t - h)]dw(t), i = 1, 2\), based on the obtained
FIGURE 1  Open-loop system state response and Markovian jump signal [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 2  Closed-loop system state response and Markovian jump signal [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 3  Aperiodic sampling control input and aperiodic sampling intervals [Colour figure can be viewed at wileyonlinelibrary.com]
TABLE 1 The obtained aperiodic sampling controllers for different $q$

<table>
<thead>
<tr>
<th>Controller</th>
<th>$q = 0$</th>
<th>$q = 1$</th>
<th>$q = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_1^{(1)}$</td>
<td>1.9855  -0.0712</td>
<td>3.7395  0.7447</td>
<td>2.5991 -0.1553</td>
</tr>
<tr>
<td></td>
<td>0.2214  2.0334</td>
<td>2.0605  3.4167</td>
<td>0.8846  2.8197</td>
</tr>
<tr>
<td>$K_1^{(2)}$</td>
<td>1.5839 -0.2095</td>
<td>1.4819 -0.1532</td>
<td>3.3548  0.5847</td>
</tr>
<tr>
<td></td>
<td>1.0220  1.9629</td>
<td>0.6179  1.5236</td>
<td>6.0955  6.4733</td>
</tr>
<tr>
<td>$K_2^{(1)}$</td>
<td>1.7680  0.0833</td>
<td>1.5307  0.0486</td>
<td>2.1896 -0.0258</td>
</tr>
<tr>
<td></td>
<td>0.3701  1.7288</td>
<td>0.2028  1.5396</td>
<td>0.8779  2.3780</td>
</tr>
<tr>
<td>$K_2^{(2)}$</td>
<td>1.5568 -0.2007</td>
<td>1.6133 -0.1673</td>
<td>2.8534  0.2120</td>
</tr>
<tr>
<td></td>
<td>0.2673  1.5601</td>
<td>0.1969  1.6040</td>
<td>4.7107  4.4569</td>
</tr>
</tbody>
</table>

FIGURE 4 Closed-loop system state response and Markovian jump signal [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 5 Aperiodic sampling control input and aperiodic sampling intervals [Colour figure can be viewed at wileyonlinelibrary.com]

sampling controller $K_1^{(1)}, q = 0$ and $K_2^{(1)}, q = 0$ to stabilize the aforementioned system, the corresponding closed-loop system response, the Markovian jump signal, the sampling control input, and the aperiodic sampling intervals are shown in Figure 4 and Figure 5. Based on the obtained sampling controller $K_1^{(1)}, q = 1$ and $K_2^{(1)}, q = 1$, the corresponding closed-loop system response, the Markovian jump signal, the sampling control input, and the aperiodic sampling intervals are shown in Figure 6 and Figure 7. Based on the obtained sampling controller $K_1^{(1)}, q = 1$ and $K_2^{(1)}, q = 1$, the corresponding closed-loop system response, the Markovian jump signal, the sampling control input, and the aperiodic sampling intervals
FIGURE 6  Closed-loop system state response and Markovian jump signal [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 7  Aperiodic sampling control input and aperiodic sampling intervals [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 8  Closed-loop system state response and Markovian jump signal [Colour figure can be viewed at wileyonlinelibrary.com]
are shown in Figure 8 and Figure 9. It is shown in these figures that the proposed method is effective in this example. We can conclude that the obtained aperiodic sampling controller based on our method can guarantee that the polytopic uncertain Itô SMJSs (1) is mean-square asymptotically stable.

### 6 CONCLUSIONS

Using parameters-dependent Lyapunov functionals and parameters-dependent time-scheduled Lyapunov functionals, two different stochastic sufficient stability criteria have been proposed for polytopic uncertain Itô SMJSs and nominal Itô SMJSs. The time-independent state feedback sampled-data controller and time-dependent state feedback sampled-data controller have been designed, respectively. Numerical simulation examples have been provided to illustrate the effectiveness of the proposed method.

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### REFERENCES


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